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CONTENTS

Pressure Waves Accompanying Detonation in the Internal Combustion Engine . . . . C. S. DRAPER 219
Boundary Layer in Compressible Fluids . . . . TH. VON KÁRMÁN AND H. S. TSIEN 227
Special Maps for Use with Loop Direction Finders . . . . JAMES M. COBURN, JR. 233
On the Stability Problem in Hydrodynamics, II . . . . C. L. PEKERIS 237
On the Triple Origin of Air Forces . . . . MAX M. MUNK 241
A Water Tank for Model Tests on the Motion of Airships in Gusts ARNOLD M. KUETHE 243
Determination of Profile Drag from Measurements in the Wake of a Body . . . . WILLIAM BOLLAY 245
Institute Notes 249
Books Received 251

Book Reviews

Handbook of Aeronautics, Vol. II: Aero Engines, Design and Practice, (*Andrew Swan*) C. FAYETTE TAYLOR 226
Aircraft Propeller Handbook, (*Karl Hansson Falk*) . . . . T. P. WRIGHT 226
Metal Airplane Structures, (*Flavius E. Loudy*); Metal Aircraft Construction, (*M. Langley*) . . . . ALEXANDER KLEMIN 232
History of Aeronautics, (*William B. Gamble*) 252
Jahrbuch der Deutschen Akademie der Luftfahrtforschung 1937–1938 252

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Pressure Waves Accompanying Detonation in the Internal Combustion Engine

C. S. DRAPER, *Massachusetts Institute of Technology*

Presented at the Power Plants and Propellers Session, Sixth Annual Meeting, I. Ae. S. January 25, 1938

Introduction

DETONATION in the internal combustion engine has been studied by many methods since the pioneer work of Midgely and Boyd[1] described in 1922. Carbon resistances,[2,3] piezoelectric crystals,[4] condensers,[5] electromagnetic generators,[6,7,8] photoelectric cells,[9] and moving mirrors have been used as sensitive elements to pick up the deflections of a diaphragm exposed to cylinder pressures. Several types of optical and electrical systems have successfully produced records from the pressure pick-up units. By means of transparent windows placed in the combustion chamber walls flame motion has been recorded,[10,11,12] combustion temperatures[13] measured, and radiant energy analyzed.[14,15] Flame movement has also been followed by means of ionization gaps exposed inside the combustion chamber.[16]

From the detonation data collected by various means a consistent theory of the process has gradually evolved. This theory holds that detonation, or knock as it is often called, is due to a very rapid pressure rise in some part of the burning gases which is followed by intense pressure waves within the charge.[4,17,18,19] Most convincing evidence of this detonation theory is given by Withrow and Rassweiler[20] who discuss results from simultaneous flame photographs and pressure records.

Despite the vast amount of time expended on the problem, very little quantitative information has appeared on the intensity of the pressure waves which accompany detonation and no systematic effort has been made to correlate observed results with charge conditions within various combustion chamber shapes. The author has devoted part of his time to this subject since 1927. A discussion of certain theoretical aspects of the problem and experimental results from two engines were published as an N.A.C.A. Report in 1934.[19] This study was handicapped by the lack of an amplifier and recording system with uniform sensitivity over a sufficiently wide frequency range, while the theoretical discussion was not given in a form suited to engineering use. Recent work on detonation at the Massachusetts Institute of Technology has been directed toward the refinement of indicating equipment and the practical application of theoretical results. The investigation is being continued with grants of funds and equipment from the N.A.C.A., the Bureau of Aeronautics of the United States Navy, the Sperry Gyroscope Co., and the Wright Aeronautical Corporation.

General Theory

Prediction of the types of pressure waves to be expected in the combustion space of an engine after knock has occurred requires fitting a solution of the wave equation for sound to the given geometrical boundaries.[21] This procedure involves assumptions that the pressure wave amplitude is small compared to the average pressure and that the gas particle velocity normal to the walls is zero. The particular case of a circular cylinder with flat ends at right angles to the

cylinder axis has been worked out by the author and applied to the C.F.R. Engine and the N.A.C.A. Universal Test Engine. The excellent checks between calculated and measured detonation frequencies certainly justify further applications of the theory. In the present paper the former results will be extended to include energy computations and the outline of a method for investigating the nature of the initial disturbance responsible for the pressure waves.

Based on the earlier work, the instantaneous pressure disturbance from equilibrium can be expressed as

$$p = \sum_{\substack{n_\theta = 0, 1, 2 \ldots \\ n_r = 0, 1, 2 \ldots \\ n_z = 0, 1, 2 \ldots}}^{\infty} p_{n_\theta n_r n_z} J_{n_\theta}[\alpha_{n_\theta n_r}(r/a)] \cos(n_\theta \theta - \varphi_{n_\theta n_r}) \times \cos(n_z \pi z/h) \cos(2\pi n_{n_\theta n_r n_z} t - \epsilon_{n_\theta n_r n_z}) \quad (1)$$

Where

- n_θ = number of azimuthal nodes (nodal planes containing the cylinder axis and a diameter).
- n_r = number of radial nodes (nodal cylinders).
- n_z = number of axial nodes (nodal planes perpendicular to the cylinder axis).
- $p_{n_\theta n_r n_z}$ = absolute amplitude of pressure associated with the mode $n_\theta n_r n_z$ (mode with nodal surfaces* $n_\theta n_r n_z$).
- $\alpha_{n_\theta n_r}$ = number associated with the nodal surfaces $n_\theta n_r$ (the method for determining these numbers is outlined in reference 19).
- r = radial distance from cylinder axis.
- a = radius of cylinder.
- θ = azimuth angle.
- $\varphi_{n_\theta n_r}$ = phase angle associated with azimuth.
- z = distance from bottom of cylinder measured parallel to the cylinder axis.
- h = height of cylinder.
- $n_{n_\theta n_r n_z}$ = frequency of the mode $n_\theta n_r n_z$.
- t = time.
- $\epsilon_{n_\theta n_r n_z}$ = phase angle associated with time.

The frequency $n_{n_\theta n_r n_z}$ is related to the velocity of sound c, and a distance $\lambda_{n_\theta n_r n_z}$ called the wavelength by the equation

$$n_{n_\theta n_r n_z} = c/\lambda_{n_\theta n_r n_z} \quad (2)$$

Sound velocity c can be found from measurements of equilibrium pressure P_0, equilibrium density ρ_0 and γ the ratio of specific heat at constant pressure to specific heat at constant volume as[19]

$$c = \sqrt{\gamma P_0/\rho_0} \quad (3)$$

Values of wavelength in terms of cylinder radius for a number of the simpler modes of vibration are given in Figs. 1, 2, and 3.

* As used in the text, the term "nodal surface" refers to a surface over which no pressure variation due to a particular mode occurs. This differs from the usage of reference 19 in which a nodal surface refers to a surface of zero particle velocity.

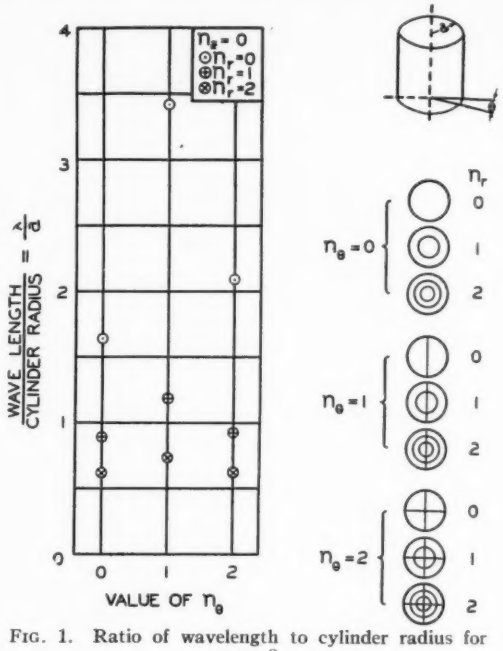


FIG. 1. Ratio of wavelength to cylinder radius for $n_z = 0$.

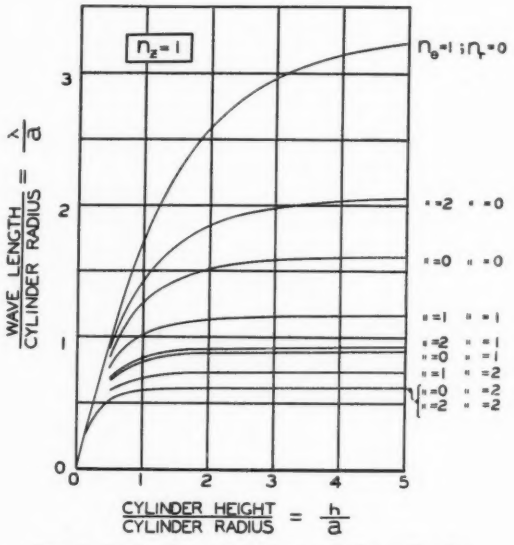


FIG. 2. Ratio of wavelength to cylinder radius for $n_z = 1$.

ENERGY CALCULATIONS

In general, the energy associated with a system of pressure waves in an enclosed volume V is

$$W = (1/\gamma P_0) \int\int\int p^2(xyz)\, dV \quad (4)$$

Where $p(xyz)$ is the pressure amplitude considered as a function of position only and the integration is extended over the entire volume. Applied to the particular case of a circular cylinder with flat ends, Eq. (4) becomes for each mode of vibration

$$W_{n_\theta n_r n_z} = \frac{p^2_{n_\theta n_r n_z}}{\gamma P_0} \int_0^{r/a=1} \int_0^{\theta=2\pi} \int_0^{z/h=1} J^2_{n_\theta}[\alpha_{n_\theta n_r}(r/a)] \cos^2 n_\theta\theta \cos^2(n_z\pi z/h) r dr d\theta dz \tag{5}$$

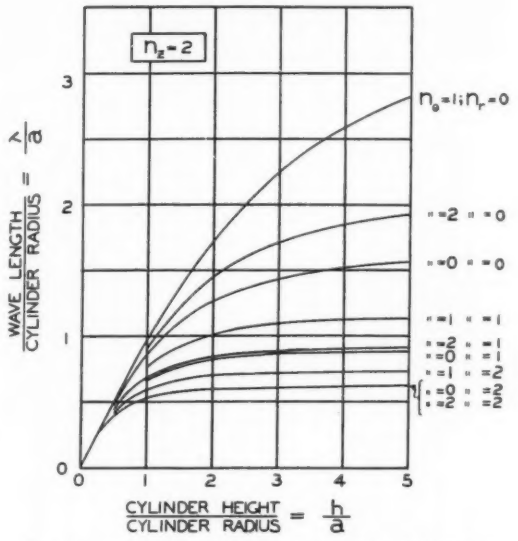


FIG. 3. Ratio of wavelength to cylinder radius for $n_z = 2$.

It is convenient to express the relationship between energy and pressures in terms of a ratio M where

$$M_{n_\theta n_r n_z} = E_{n_\theta n_r n_z} / (p^2_{n_\theta n_r n_z} / \gamma P_0) \tag{6}$$

In this equation $E_{n_\theta n_r n_z}$ is the average energy density associated with the mode $n_\theta n_r n_z$.

The ratio M is given for a number of modes in column 4 of Table 1. Column 5 of this table is the calculated frequency corresponding to these same modes of vibration based on typical conditions existing near top center in a C.F.R. Engine with a compression ratio of 4.85.

TABLE 1
Based on Sound Velocity of 3000 ft./sec. in C.F.R. Engine.

Azimuthal Nodes n_θ	Radial Nodes n_r	Axial Nodes n_z	$M_{n_\theta n_r n_z}$	Cycles/sec. $n_{n_\theta n_r n_z}$
0	0	0	1.000	13540
0	0	1	.500	19650
0	0	2	.500	30830
0	1	0	.1620	24900
0	1	1	.0810	29200
0	1	2	.0810	37000
0	2	0	.0903	35900
0	2	1	.0452	39000
0	2	2	.0452	46300
1	0	0	.119	6500
1	0	1	.0595	15420
1	0	2	.0595	28900
1	1	0	.0537	18800
1	1	1	.0269	23650
1	1	2	.0269	34200
1	2	0	.0368	30100
1	2	1	.0814	34200
1	2	2	.0814	41900
2	0	0	.0332	10620
2	0	1	.0166	17780
2	0	2	.0166	29650
2	1	0	.0450	24000
2	1	1	.0225	28200
2	1	2	.0225	36400
2	2	0	.0318	35800
2	2	1	.0159	39000
2	2	2	.0159	46300
3	0	0		14700
3	1	0		28400
4	0	0		19000
4	1	0		33100
5	0	0		22900
5	1	0		37000
6	0	0		26300
6	1	0		43300
7	0	0		30000
8	0	0		33800
9	0	0		37700

PRESSURE WAVES DUE TO GIVEN INITIAL KNOCK

The type and amplitude of the vibrations produced by a given knock will depend upon the shape, location, and intensity of the initial disturbance. For computation it is assumed that this disturbance can be approximated by assigning a constant pressure p_0 to a region between r_1 and a, 0 and z_1, $-\theta_1$ and θ_1. If both sides of Eq. (1) are multiplied by the variables on the right-hand side and integration over the whole cylinder volume is carried out for $t = 0$, an expression for the pressure amplitude of each mode of vibration is obtained as

$$p_{n_\theta n_r n_z} = \frac{\int_{r_1/a}^{1} \int_{-\theta_1}^{\theta_1} \int_0^{z_1/a_1} p_0 J_{n_\theta}[\alpha_{n_\theta n_r}(r/a)] \cos(n_\theta\theta - \varphi_{n_\theta n_r}) \cos(n_z\pi z/h) r dr d\theta dz}{\cos \epsilon_{n_\theta n_r n_z} \int_0^1 \int_0^{2\pi} \int_0^1 J^2{}_{n_\theta}[\alpha_{n_\theta n_r}(r/a)] \cos^2(n_\theta\theta - \varphi_{n_\theta n_r}) \cos^2(n_z\pi z/h) r dr d\theta dz} \tag{7}$$

The integral in the denominator has already been evaluated in connection with the energy factors of Eq (6). Using this result the pressure amplitudes can be written

$$p_{n_\theta n_r n_z} = \frac{p_0}{\cos \epsilon_{n_\theta n_r n_z} M_{n_\theta n_r n_z} V} \int_{r_1/a}^{1} \int_{-\theta_1}^{\theta_1} \int_{0}^{z_1/h} J_{n_\theta}[\alpha_{n_\theta n_r}(r/a)] \cos(n_\theta \theta - \varphi_{n_\theta n_r}) \cos(n_z \pi z/h) r dr d\theta dz \tag{8}$$

where V = volume of charge.

The integrals of Eq. (8) are somewhat difficult to calculate but the work has already been carried out for several cases. Preliminary checks with experimental results have shown qualitative agreement but a detailed discussion of this subject is beyond the scope of the present paper.

Experimental Method

The ultimate object of the M.I.T. Detonation Research is to produce an instrument suitable for quantitative measurements on any type of engine under any operating conditions as well as in the laboratory. The first step in this program was to construct laboratory apparatus to record accurately the physical phenomena available to operate an instrument in routine work. For the present case a reasonably certain check between theory and experiment required at least two simultaneous records of detonation waves in order to measure phase and amplitude relations. With the amplitudes of pressure waves small compared to total cylinder pressures, it is difficult to study the wave motion directly from a pressure card. Because of the emphasis on the pressure waves found in a rate of change of pressure record,[6,8,19] the author chose to continue the development of an electromagnetic instrument which gives an output directly proportional to the rate of change of pressure.

In order to insure that no essential part of the detonation process is distorted the indicator must give true records of pressure variations from those of the ordinary indicator card up to frequencies of about 25,000 cycles per sec. This requires a diaphragm natural frequency in the vicinity of 100,000 cycles per sec. in order to prevent errors from mechanical effects.[22] In considering errors of this type the diaphragm can be considered as substantially undamped. A second advantage of using a high natural frequency for the diaphragm is that the effects of engine vibration are minimized.

Pick-Up Units

Fig. 4 shows the essential parts of two types of electromagnetic pick-up units. In both cases the necessary magnetic field is supplied by a central core of heat-treated cobalt steel. The right-hand unit has an annular air gap with a cylindrical inductor machined as an integral part of the diaphragm. Any deflection will cause a transfer of flux from one side to the other of the stationary coil and thus produce an induced voltage. The left-hand unit also has a stationary generating coil

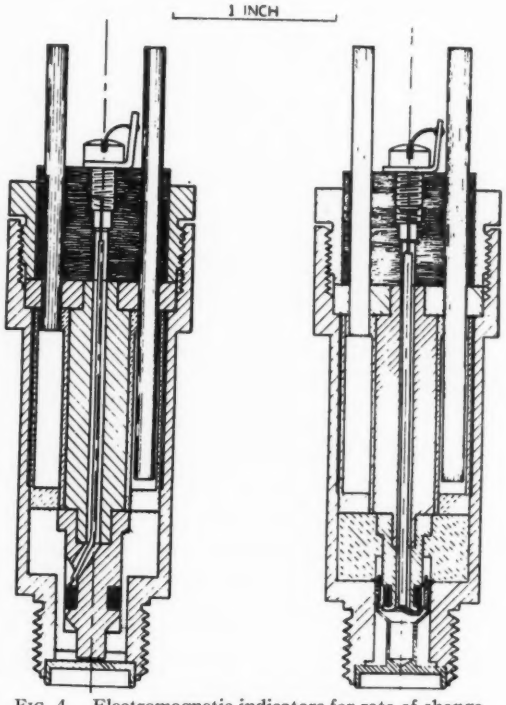


Fig. 4. Electromagnetic indicators for rate of change of pressure.

wound on the central core and the flux changes are due to changes in the air gap between the pole piece and the diaphragm in the manner of the conventional telephone receiver. A water jacket around the magnet is shown in both units. Many hours of operation in various engines have shown that the water jacket is unnecessary in practice. The permanent magnet seems to retain its strength satisfactorily after an initial "shakedown" period.

It is possible to construct a very sensitive unit using a cylindrical inductor but the natural frequency is of necessity much lower than that possible with the flat diaphragm type. This natural frequency advantage makes the telephone receiver type more desirable than the other instrument. The diaphragm in use at the present time is 0.060 in. in thickness and 0.500 in. in diameter which gives a natural frequency of about 95,000 cycles per sec. Photographs of the actual instrument are shown in Fig. 5.

Amplifier-Oscillograph System

The wide frequency range required makes it necessary to use cathode ray oscillographs for the indicating elements. For satisfactory photographic records with the

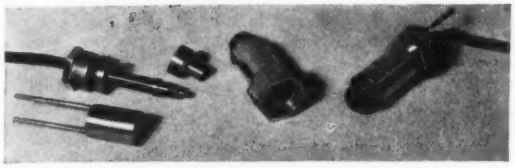


FIG. 5. Photograph of rate of change of pressure indicator.

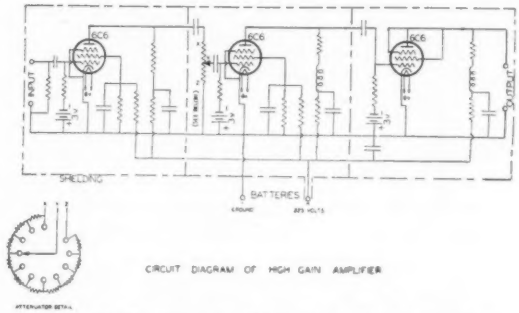
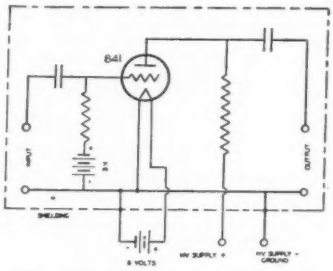


FIG. 6. Wiring diagram of high gain section of amplifier.



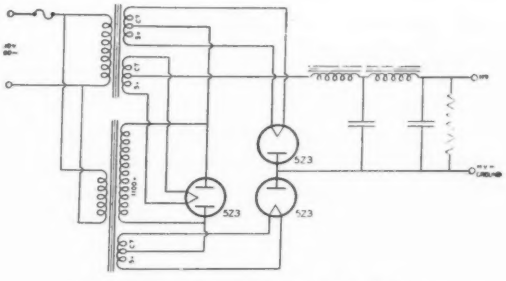


FIG. 7. Wiring diagram of high voltage amplifier and power supply.

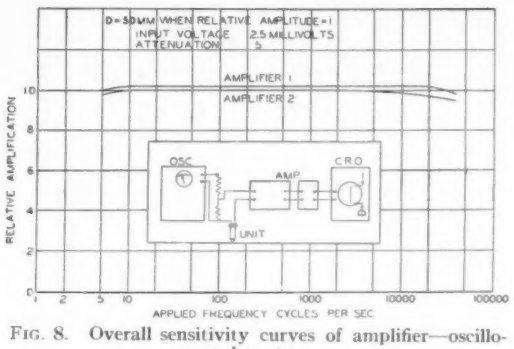


FIG. 8. Overall sensitivity curves of amplifier—oscillograph systems.

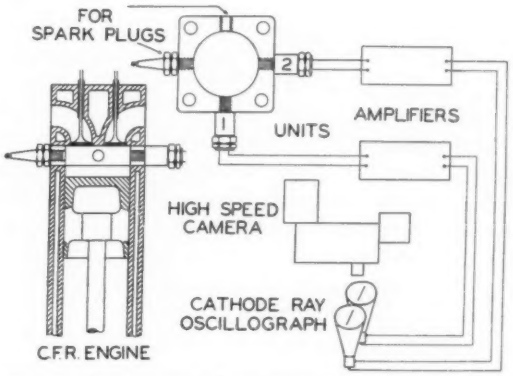


FIG. 9. Arrangement of apparatus for study of detonation in the C.F.R. Engine.

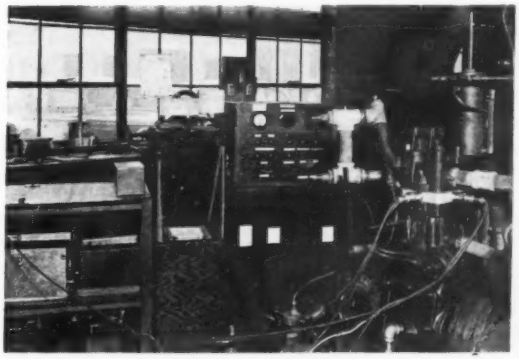


FIG. 10. Photograph of apparatus for recording detonation in the C.F.R. Engine.

film speed of 400 in. per sec. a specially designed power supply of 4000 volts was used. The amplifiers were specially designed in two sections, a high-gain section connected between the indicators and a high-voltage section to operate the cathode ray tube. Figs. 6 and 7 show wiring diagrams of the amplifier units. Sensitivity curves for the two units are given in Fig. 8.

ENGINE TEST

The general arrangement of apparatus for tests on a C.F.R. Engine is indicated in Fig. 9. A special spacer between the cylinder and the head was used to permit the location of indicators and spark plugs in four alternate positions. Fig. 10 shows the actual appearance of the setup.

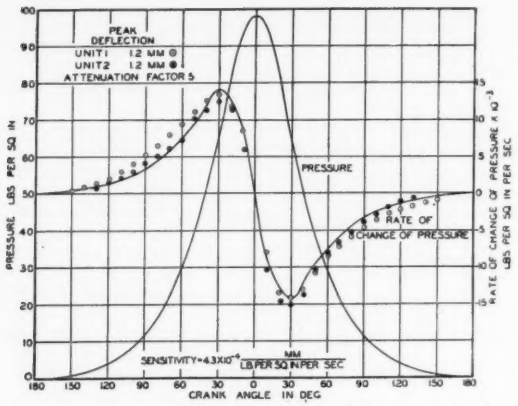


FIG. 11. Indicator calibration by non-firing operation in the C.F.R. Engine.

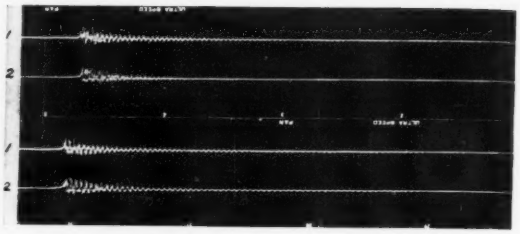


FIG. 12. Typical records of two successive cycles in the C.F.R. Engine. Attenuation factor = 200. Speed 1230 r.p.m., spark advance 39°; inlet temperature 150°F.; 72 octane fuel; moderate detonation induced by ethyl nitrite; mixture ratio 14:1; air consumption 1.13×10^{-3} lbs. per cycle.

CALIBRATION

Overall calibrations of the apparatus are carried out by comparing indicator records from the apparatus with a rate of change card derived from an M.I.T. Pressure Indicator record.[23] Fig. 11 shows the result of such a calibration. By this method the maximum sensitivity was found to be 4.3×10^{-4} mm. per lb. per sq.in. per sec. This value can be varied over a range of one thousand to one by means of the attenuators.

TYPICAL RECORDS

Fig. 12 shows records from successive cycles in the C.F.R. Engine with moderate detonation as judged by ear. Tests under identical operating conditions with blind plugs shielding the indicators from cylinder gases showed that direct vibration effects were negligible. Records taken at the same engine speed but without audible detonation were substantially free from vibration during the power stroke.

OBSERVED FREQUENCIES

The most prominent frequency shown in Fig. 12 is near 6500 cycles per sec. and decreases as the piston descends. This value agrees with results already published by the writer and other investigators.[5,8,19] By reference to the last column of Table 1, which is based upon a sound velocity corresponding approximately to top center power stroke conditions in the C.F.R. Engine, the mode of vibration excited most strongly has one azimuthal node and no axial or radial nodes. Fig. 13 shows the lines of constant pressure amplitude for waves of this type. Dotted lines on one side of the nodal plane indicate the same time phase while full lines on the other side of the node show the opposite phase. It is apparent that indicators located at positions A_1 and A_2 will show substantially equal amplitudes of opposite phase so far as this particular mode of vibration is concerned. Indicators located at B_1 and B_2 will show a strong vibration and no vibration, respectively, from this same mode. Thus the upper record of Fig. 12 which shows very little of the lowest frequency mode at unit 2 can be explained on the assumption that one end of the pressure node was located immediately in front of this indicator. The lower record of Fig. 12 in which the vibrations are out of phase corresponds to the case with indicators located symmetrically on either side of the nodal plane.

High-frequency vibrations are present on all the records especially in the initial stages. An extended examination of many records from various engines has shown one vibration component of about 50,000 cycles per sec. frequency which can possibly be attributed to longitudinal vibrations of the indicator pole piece. However, there are definitely other frequencies present corresponding to more complicated modes of vibration as shown in Table 1. Quantitative work on these modes would require an increase in film speed of the recording apparatus.

PRESSURE WAVE ENERGY

One of the most serious difficulties in measuring the physical quantities associated with detonation is due to the relatively great variations between successive cycles even under carefully adjusted engine conditions. Until engine operation can be more precisely controlled any estimate of the energy associated with detonation pressure waves must be considered as only somewhat better than an order of magnitude rather than a precise value. With this in mind the wave energy associated with the cycle recorded in the lower traces of Fig. 12 will be calculated.

To determine the pressure amplitude associated with each component taken from the records, it is necessary to divide the measured rate of change amplitude by 2π times the corresponding frequency. This means that the actual pressures associated with high-frequency components will be small unless the record amplitudes are large. In the present case only the pressure waves of the lowest frequency in Fig. 12 will make any important contribution to the energy. Estimating from the record that the maximum rate of change amplitude near 10° from top center is 2 mm., for the lowest fre-

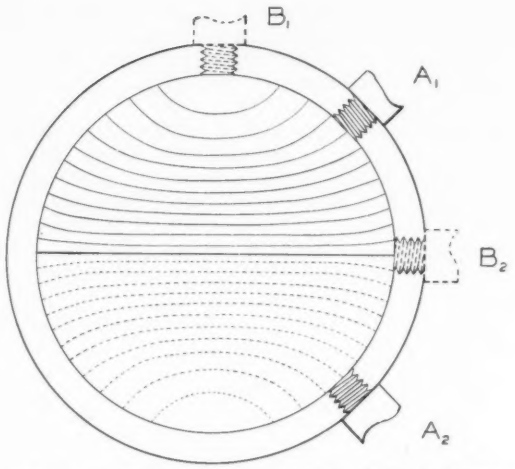


FIG. 13. Lines of equal pressure amplitude for the lowest frequency mode of vibration in a circular cylinder.

quency mode, the pressure amplitude will be about 23 lbs. per sq.in. The measured pressure for this crank angle was about 350 lbs. per sq.in. and the temperature about 4000°F. absolute as taken from the Hottel chart.[24] This corresponds to a ratio of specific heats of about 1.24.[19] From Table 1 the value of M_{100} is 0.119. Substituting these values in Eq. (6) and solving for E_{100} gives an average energy density associated with the pressure waves of 1.7 ft. lbs. per cu.ft. In the C.F.R. Engine at 10° after top center this corresponds to a total energy of 0.01 ft. lbs. (1.3×10^{-5} B.t.u.) The measured weight of air in the theoretical mixture per cycle was 1.13×10^{-3} lbs. which gives a total of about 1.38 B.t.u. chemical energy per cycle[24] or an available energy of 0.35 B.t.u. if a thermal efficiency of 25 percent is assumed. This indicates that less than 0.01 percent of the available charge energy is involved in the pressure waves accompanying detonation. This does not mean that only this small fraction of the charge explodes suddenly in the initial disturbance since the efficiency of converting chemical energy into sound energy is probably low. E. S. Taylor has given an estimate for a theoretical cycle with instantaneous combustion of the detonating part of the charge limiting the highest possible efficiency for sound wave production at 13 percent, for a fuel-air ratio of .078 and a compression ratio of 6. No doubt combustion is sufficiently slow so that the true value is much less. Even after a factor of this magnitude is applied, the amount of charge required to produce observed detonation effects will be a fraction of one percent.

DISCUSSION AND CONCLUSIONS

The general experimental conclusion that the energy associated with pressure waves due to detonation is a small percentage of the total energy falls nicely into agreement with the known result that the output of a well-designed engine is not affected by moderate detonation intensities.

Since the actual wave pressures are too small to directly damage engine parts, other effects must be considered. Withrow and Rassweiler have advanced the theory that in detonation " . . . . . *The increased jacket temperature and loss of power both undoubtedly result from increased heat transfer from gases to the walls. It is apparent that this heat transfer would be greatly augmented by the 'scrubbing action' of the high gas velocities induced by knock . . . . . .*"[12]

With the present data it is possible to calculate the maximum particle velocity at the cylinder wall due to the measured pressure waves. In general the particle velocity, v, due to a sinusoidal pressure wave in a gas is[21]

$$v = -(\nabla p_a/\rho_0 2\pi n)\sin 2\pi nt \qquad (9)$$

where p_a is the space distribution of pressure amplitude.

Substituting values for the lowest frequency mode as given by the records of Fig. 9 and remembering that the velocity normal to the cylinder wall must be zero gives a maximum particle velocity of 50 ft. per sec. While this figure does not represent a very high speed, the oscillatory motion will take place very close to the cylinder wall and possibly supply the "scrubbing action" suggested by Withrow and Rassweiler. This measured gas velocity due to pressure waves will certainly be intensified for surfaces near the openings of pockets due to alternating flow in and out with a resultant heating of spark plug points and similar parts. In addition, at the instant of the initial explosion, all the energy which is later spread throughout the cylinder volume will be concentrated in a relatively small region with the result of very high temperatures, densities, and surface velocities as pressure waves spread over the cylinder. The attendant high rate of heat transfer can well be responsible for the local pitting effects observed in the aluminum combustion chamber surfaces of certain engines.

In regard to the problem of selecting a unit of detonation intensity suitable for all engines under all conditions, it seems reasonable to use the average pressure wave energy density as the quantitative measure. From Eq (5) it is seen that this energy density is proportional to the square of the pressure wave amplitude. Thus if an electromagnetic generator type of indicator is used, the associated electrical circuit should first integrate to reduce all frequencies to a common basis, then "rectify" and amplify according to a square law with a final steady indication proportional to the rectified output. This general problem is now being studied and it is hoped to arrive at a satisfactory answer in the not too distant future.

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Book Reviews

Handbook of Aeronautics, Vol. II: Aero Engines, Design and Practice, by ANDREW SWAN; Pitman Publishing Corporation, New York, 1938; 483 pages, ill., $7.50.

As in previous editions (1931, 1934) this book consists of a more or less complete compilation of descriptive matter, plus a considerable amount of statistical data, concerning British aeronautical power plants. Foreign practice is also included, though to a much more limited extent. The material is largely obtained from published sources, very little original data being presented. In addition to chapters on engines and their component parts, chapters are included on supercharging, fuel systems, lubrication, exhaust systems, heaters, starters, and engine testing. A large number of performance curves are presented, both for engines and their auxiliaries, together with tables of the "standard" altitude, viscosity conversion charts, physical and thermal properties of materials, etc.

In general, the arrangement is good and the presentation clear, which results in a volume of considerable value for reference purposes. On the other hand, publications of this kind invariably suffer from the fact that vital statistics on aviation power plants are seldom available until the models in question are virtually obsolete. For example, no rating for the Wright "Cyclone" in excess of 768 hp. is given, and the table on bearing loads (page 28) includes such obsolete and unsuccessful examples as the U. S. A. W-1, the Packard 2500, etc. Fortunately, the British engines used for illustration are somewhat more up-to-date.

The chief fault of this book, however, is the notable absence of critical appraisal of designs and design methods, practice both good and bad being presented without comment from the author. No advantage is taken of the fundamental principals of similitude, and thus very few of the data are reduced to those non-dimensional ratios which must be the basis of any intelligent new design. While this book constitutes a welcome addition to the designer's library, one is reluctantly forced to the conclusion that a definitive work on aero-engine design has yet to be written.

C. FAYETTE TAYLOR

Aircraft Propeller Handbook, by KARL HANSSON FALK; The Ronald Press Company, New York, 1937; 99 pages, ill., $4.00.

The first paragraph of the author's preface reads as follows:

"This AIRCRAFT PROPELLER HANDBOOK has been prepared to meet the needs of engineers, draughtsmen, students, pilots, service personnel, and others in the aeronautic industry who wish to obtain concise, practical data without delving into propeller theory."

This quotation very adequately describes the Propeller Handbook which Mr. Falk has produced. Rules and formulae for determining basic propeller characteristics are given, supplemented by numerous charts, curves, and figures. Starting out with the main characteristics necessary from a propeller design standpoint, the author follows with concise information on static, gyroscopic, and aerodynamic characteristics; a chapter on twisting moments and strength, and then some very interesting information on vibration. He then proceeds with citation of several examples to illustrate the uses of the formulae and curves of the Handbook, with a final chapter on method of approach for a designer to use in the selection of a propeller for his airplane.

This Handbook gives every evidence of being a careful rearrangement of the running notes used by a practical designer, which is just what the author is, and therefore it appears certain that this small propeller handbook will be found extremely useful for the practical man who does not care to delve through the more complete textbooks or theoretical treatises on propeller design in order to have available concise information for immediate use.

T. P. WRIGHT
Curtiss-Wright Corporation

Boundary Layer in Compressible Fluids

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Presented at the Aerodynamics Session, Sixth Annual Meeting, I. Ae. S.
January 26, 1938

SUMMARY

The first part of the paper is concerned with the theory of the laminar boundary layer in compressible fluids. The known solution for incompressible fluids is extended to large Mach's numbers by successive approximation. The compressibility effect on surface friction is discussed, and the results applied to estimate the ratio between wave resistance and frictional drag of projectiles and rockets. In the second part the heat transfer between a hot fluid and a cool surface, then between a hot body and a cool fluid is discussed. A general relation between drag and heat transfer as function of Mach's number is given. The limits where cooling becomes illusory because of the heat produced by friction are determined.

THE solution of flow problems in which the density is variable is in general very difficult; hence, every case in which an exact or even an approximate solution of the equations of the motion of compressible fluids can be obtained has considerable theoretical interest. Several authors noticed that the theory of the laminar boundary layer can be extended to the case of compressible fluids moving with arbitrarily high velocities without encountering insurmountable mathematical difficulties. Busemann[1] established the equations and calculated the velocity profile for one speed ratio. (By speed ratio is understood the ratio of the airspeed to the velocity of sound.) Frankl[2] also made an analysis of the same problem, however, it is complicated and depends on several arbitrary approximations. The senior author[3] obtained a first approximation by a simple but apparently not sufficiently exact calculation. Hence, in the first part of the present paper, a better method for the solution of the problem is developed.

The boundary layer theory for very high velocities is not without practical interest. First, the statement can be found often in technical and semi-technical literature on rockets and similar high-speed devices that the skin friction becomes more and more insignificant at high speeds. Of course, it is known that with increasing Reynolds Number, the skin friction coefficient is decreasing, *i.e.*, the skin friction becomes relatively small in comparison with the drag produced by wave formation or direct shock. Since high-speed flight will be performed mostly at high altitude where the air is of very low density, so that the kinematic viscosity is large, the resulting Reynolds Number will be relatively small in spite of the high speed.

Another interesting point in the theory of the boundary layer in compressible fluids is the thermodynamic aspect of the problem. In the case of low speeds the influence of the heat produced in the boundary layer can be neglected both in the calculation of the drag and of the heat transferred to the wall. In the case of high speeds, however, the heat produced in the boundary layer is not negligible, but determines the direction of heat flow. In the second part of the paper a few simple examples of heat flow through the boundary layer are discussed.

It has been found necessary in most parts of this analysis to make the assumption of laminar flow. This assumption was found necessary because of the lamentable state of knowledge concerning the laws of turbulent flow of compressible fluids at high speeds. This assumption is somewhat justified by the fact that—as mentioned above—in many problems where the results of this paper can be applied, the Reynolds Number is relatively small, so that a considerable portion of the boundary layer is probably, *de facto*, laminar. Ackeret[4] called attention to the possibility that the stability conditions in supersonic flow might be quite different from those occurring in flow with low velocities. The authors also believe that the stability criteria as developed by Tollmien and others, cannot be applied without modification. Finally, some conclusions of the paper, as will be pointed out, are also applicable to turbulent flow. In other cases, as in the calculation of drag, the assumption of laminar flow surely gives at least the lower limit of its value.

I

If the x-axis is taken along the plate in the direction of the free stream, the y-axis perpendicular to the plate, and u and v indicate the x and y components of the velocity at any point, then the simplified equation of motion in the boundary layer is

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \tag{1}$$

where both the density ρ and the viscosity μ are variables.

The equation of continuity in this case is

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{2}$$

A third equation determines the energy balance between the heat produced by viscous dissipation and the

heat transferred by conduction and convection. With the same simplification as used in Eqs. (1) and (2), one can write

$$\rho u \frac{\partial}{\partial x}(c_p \cdot T) + \rho v \frac{\partial}{\partial y}(c_p \cdot T) = \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \mu \left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

where c_p is the specific heat at constant pressure, and λ is the coefficient of heat conduction. If Prandtl's number, $c_p\mu/\lambda$ is assumed to be equal to 1, then it can be easily shown that both Eqs. (1) and (3) can be satisfied by equating the temperature T to a certain parabolic function of the velocity u only. This relation between T and u is

$$\frac{T}{T_0} = \frac{T_w}{T_0} - \left(\frac{T_w}{T_0} - 1\right)\frac{u}{U} + \frac{\kappa - 1}{2} M^2 \frac{u}{U}\left(1 - \frac{u}{U}\right) \tag{4}$$

where

U = free stream velocity.
M = speed ratio, or Mach's number of the free stream.
T_0 = temperature of the free stream.
T_w = temperature at the wall of the plate.

Differentiating Eq. (4) one obtains

$$\frac{1}{T_0}\left(\frac{\partial T}{\partial y}\right)_w = \frac{1}{U}\left[\frac{\kappa - 1}{2} M^2 - \left(\frac{T_w}{T_0} - 1\right)\right]\left(\frac{\partial u}{\partial y}\right)_w \tag{5}$$

where the subscript w refers to conditions existing at the surface of the plate. Now $(\partial u/\partial y)_w$ is always positive; therefore, if $[(\kappa - 1)/2]M^2 > (T_w/T_0) - 1$ heat is transferred from the fluid to the wall, if $[(\kappa - 1)/2]M^2 = (T_w/T_0) - 1$ there is no heat transfer between the fluid and the wall, and if $[(\kappa - 1)/2]M^2 < (T_w/T_0) - 1$ heat is transferred from the wall to the fluid. If there is no heat transfer, the energy content per unit mass $(u^2/2) + c_pT$ is constant in the whole region of the boundary layer.[5,6]

The pressure being constant the relation between ρ and T is,

$$\rho = \rho_0 \frac{T_0}{T} \tag{6}$$

The expression for the viscosity based on the kinetic theory of gases is

$$\mu = \mu_0 (T/T_0)^{1/2} \tag{7}$$

However, the following formula is in closer agreement with experimental data

$$\mu = \mu_0 (T/T_0)^{0.76} \tag{7a}$$

Busemann[1] calculated the limiting case for which $[(\kappa - 1)/2]M^2 = (T_w/T_0) - 1$ using Eq. (7) and found that for a high Mach's number, the velocity profile is approximately linear. The senior author,[3] using this linear velocity profile, the integral relation between the friction and the momentum, and Eq. (7) found that

TABLE 1

M	0	1	2	5	10	∞
Θ	1.16	1.20	1.25	1.39	1.50	1.57

$$C_f = \frac{\textit{Frictional force per unit width of plate}}{(\rho_0 U^2/2) \times \text{length of plate}} = \Theta \sqrt{\frac{\mu_0}{\rho_0 U x}} \left\{1 + \frac{\kappa - 1}{2} M^2\right\}^{-1/4} \tag{8}$$

The dimensionless quantity Θ shown in Table 1 is a function of Mach's number only.
However, if Eq. (7a) is used, then

$$C_f = \Theta \sqrt{\frac{\mu_0}{\rho_0 U x}} \left\{1 + \frac{\kappa - 1}{2} M^2\right\}^{-0.12} \tag{8a}$$

It is evident that this linear approximation is not satisfactory for small values of Mach's number. For $M = 0$, the case is the same as the Blasius solution[7] for incompressible fluids for which Θ is 1.328.

To solve the problem more rigorously, one has to resort to Eqs. (1) and (2). By introducing the stream function ψ which is defined by

$$\frac{\rho}{\rho_0} u = \frac{\partial \psi}{\partial y}, \quad -\frac{\rho}{\rho_0} v = \frac{\partial \psi}{\partial x}$$

the equation of continuity, Eq. (2), is satisfied automatically. Now, if in Eq. (1) ψ is introduced as the independent variable as was done by von Mises[8] in his simplification of the boundary layer equation for incompressible fluids, and all terms are reduced to non-dimensional form then

$$\frac{\partial u^*}{\partial n^*} = \frac{\partial}{\partial \psi^*}\left(u^* \rho^* \mu^* \frac{\partial u^*}{\partial \psi^*}\right) \tag{9}$$

where

$$\begin{aligned} u^* &= u/U \\ n^* &= n/L \\ \psi^* &= (\psi/UL)\sqrt{\rho_0 UL/\mu_0} \\ \rho^* &= \rho/\rho_0 \\ \mu^* &= \mu/\mu_0 \end{aligned} \tag{9a}$$

and L is a convenient length, say length of the plate.

Eq. (9) can be further simplified by introducing a new independent variable $\zeta = \psi^*/\sqrt{n^*}$, then

$$-\frac{\zeta}{2}\frac{du^*}{d\zeta} = \frac{d}{d\zeta}\left(u^* \rho^* \mu^* \frac{du^*}{d\zeta}\right) \tag{10}$$

This can be solved by the method of successive approximations. As ρ^* and μ^* are functions of tempera-

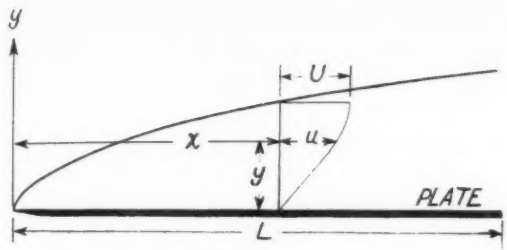

FIG. 1.

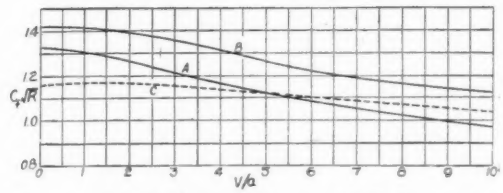

FIG. 2. Skin friction coefficients. (a) No heat transferred to wall. (b) Wall temperature 1/4 of free stream temperature. (c) von Kármán's first approximation.

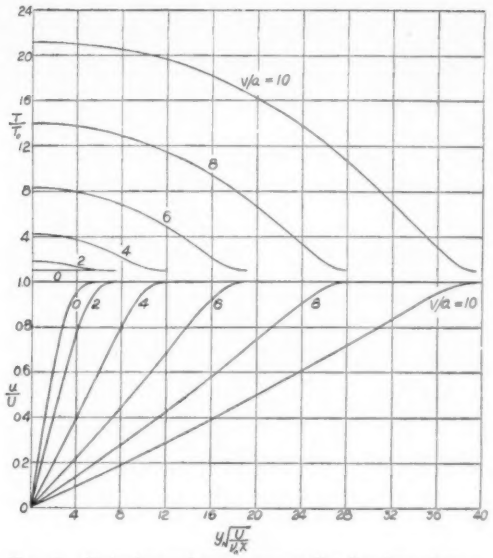

FIG. 3. Velocity and temperature distribution when no heat is transferred to wall.

ture only as shown in Eqs. (6) and (7) or (7a) and the temperature is a function of u^* then by starting with the known Blasius' solution[6] the right-hand side of Eq. (10) can be expressed in terms of ζ. Therefore, one can write

$$u^*\rho^*\mu^* = f(\zeta)$$

Consequently, the solution of Eq. (10) is

$$u^* = C \int_0^\zeta \frac{F}{f}\, d\zeta \tag{11}$$

where

$$F = exp\left(-\int_0^\zeta \frac{\zeta d\zeta}{f}\right)$$

and C is determined by the boundary condition,

$$\frac{1}{C} = \int_0^\infty \frac{F}{f}\, d\zeta \tag{11a}$$

A second approximation can be made based upon the value of u^* obtained from Eq. (11). It has been found in the cases investigated that the third or fourth approximation gives sufficient accuracy.

Having computed the final u^*, the y corresponding to u^* can be calculated from

$$y\sqrt{U\rho_0/\mu_0 x} = \int_0^\zeta d\zeta/\rho^* u^* \tag{12}$$

Also the skin friction can be computed by the momentum law,

$$C_f = \frac{F}{\frac{\rho_0}{2}U^2 L} = \frac{2\int_0^\infty (1 - u^*)d\zeta}{\sqrt{R}} \tag{13}$$

The velocity profile, the temperature distribution, and the frictional drag coefficient are calculated for different values of the Mach's number of the free stream, for the case $[(\kappa - 1)/2]M^2 = (T_w/T_0) - 1$ using the approximate viscosity relation of Eq. (7a). The results are shown in Figs. 2 and 3. The velocity profiles for high speeds are very nearly linear, but it can be seen that the wall temperature for greater Mach's numbers is very high. If the free stream temperature is 40°F., then the wall temperature will be 1600°F., 3620°F., 6540°F., and 10,170°F. for Mach's numbers of 4, 6, 8, and 10, respectively. Therefore, there is no doubt that the law of viscosity as expressed by Eq. (7a) will not hold. Also at such high temperatures, the heat transfer due to radiation cannot be neglected. Therefore, the results for extreme Mach's numbers are qualitative only.

The change in the constant $C_f\sqrt{R}$ is appreciable, but not great. It decreases from 1.328 for $M = 0$ to 0.975 for $M = 10$, or about 30 percent. However, for $0 < M < 3$ the change of the constant is very small.

Fig. 2 also shows that Eq. (8a) which was obtained by using the linear approximation is fairly accurate for very high Mach's numbers.

As examples, consider first a projectile and second, a wingless sounding rocket. Taking the diameter of the projectile to be 6 in., the length 24 in., the velocity 1500 ft./sec. and the altitude 32,800 ft. (10 km.), then the Reynolds Number based on the total length is 7.86×10^6 and the speed ratio is 1.52. From Fig. 2 the skin friction coefficient is

$$C_f = (1.286 \times 10^{-3})/\sqrt{7.86} = 0.000459.$$

Changing the skin friction coefficient (based on the skin area) to the drag coefficient (based on the maximum cross-section), one obtains

$$C_{D_f} = 0.0055.$$

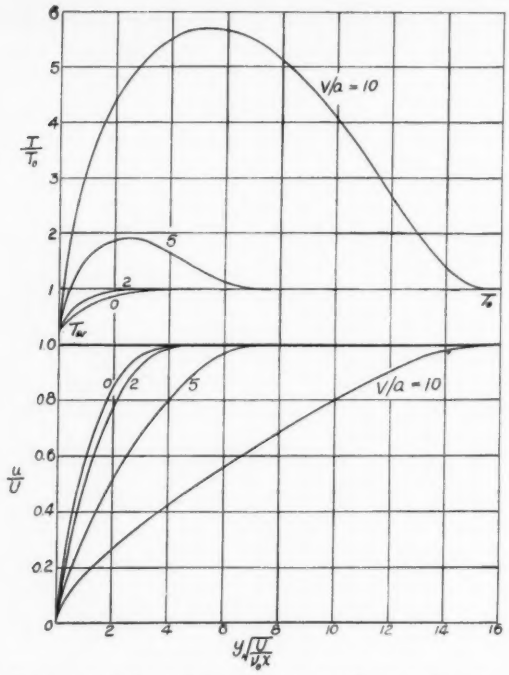

FIG. 4. Velocity and temperature distribution when the wall temperature is $^1/_4$ of the free stream temperature.

The drag coefficient due to wave formation taken from Kent's experiments[9] is

$$C_{D_w} = 0.190.$$

Therefore, the ratio of skin friction to wave resistance is 0.0055/0.190 = 0.029.

However, the ratio is greatly changed in the case of the rocket. Taking the diameter of the rocket to be 9 in., the length 8 ft., and the altitude of flight 50 km.* (164,000 ft.), the velocity 3400 ft./sec., then the Reynolds Number based on a density ratio at that altitude of 0.00067 and temperature 25°C. (deduced from data on meteors) is 6.14×10^5, and the speed ratio is 3.00. From Fig. 2, the skin friction coefficient is

$$C_f = (1.213 \times 10^{-2})/\sqrt{11.4} = 0.00360.$$

Then

$$C_{D_f} = 0.123.$$

The drag coefficient due to wave formation from Kent's experiments[9] is

$$C_{D_w} = 0.100.$$

Therefore, the ratio of skin friction and wave resistance is now 0.123/0.100 = 1.23. If the boundary layer is partly turbulent, the ratio will be even greater. This shows clearly the importance of skin friction in the case of a slender body moving with high speed in extremely rarefied air. It also disproves the belief that wave resistance would always be the predominating part in the total drag of a body moving with a velocity higher than that of sound. The reason underlying this fact can be easily understood when one recalls that the wave resistance of a body is approximately directly proportional to the velocity, while the skin friction is proportional to the velocity raised to a power between 1.5 and 2. Therefore, the ratio of skin friction to wave resistance increases with the speed. With very high velocities and high kinematic viscosity, the wave resistance may even be a negligible portion of the total drag of the body.

II

In order to point out the thermodynamic aspect of the problem two cases will be considered: the flow of a hot fluid along a surface which is kept at a constant temperature inferior to that of the fluid, and the case of a hot wall cooled by a fluid of lower temperature. The problems treated in this part have been discussed before in two very interesting papers by L. Crocco.[5,6] He especially gives an elegant treatment of the cooling problem in the case of very high velocities ("Hyperaviation"). The authors feel that their treatment is somewhat more general and extended than Crocco's previous analysis.

An interesting general relation between the heat transferred through the wall and the frictional drag can be obtained using the assumption that Prandtl's number, *i.e.*, the ratio $c_p\mu/\lambda$, is equal to unity. The same assumption was used also in the previous calculations. It is remarkable that the relation holds also as well for laminar as for turbulent flow. The heat flow q per unit time and unit area of the wall surface is

$$q = \lambda_w(\partial T/\partial y)_w$$

and the frictional drag τ per unit area is

$$\tau = \mu_w(\partial u/\partial y)_w$$

Using Eq. (4) the ratio q/τ can be calculated from the relation

$$\frac{q}{\tau} = \frac{\lambda_w}{\mu_w}\frac{T_0}{U}\left[\left(1 - \frac{T_w}{T_0}\right) + \frac{\kappa - 1}{2}M^2\right] \quad (14)$$

where T_0 is the absolute temperature, and U the velocity of the fluid in the free stream, T_w the absolute temperature at the wall, λ_w and μ_w are the heat conduction and

* The hydrodynamic equation holds so long as the mean free path of the molecules is small in comparison with the thickness of the boundary layer. For this case the thickness of the boundary layer is zero at the nose, however, at a distance $^1/_4$ of the length of the rocket it already amounts to 3.2 cm., while the calculated mean free path of the air molecules at the altitude considered is about 1.1×10^{-2} cm. Hence it appears that even for this case the theory can be safely applied. This conclusion is substantiated by the experimental results of H. Ebert in "*Darstellung der Strömungsvorgänge von Gasen bei neidrigen Drucken mittels Reynoldsscher Zahlen*," Zeitschrift für Physik, Bd. 85, S. 561-564, 1933.

viscosity coefficients of the fluid corresponding to the wall temperature and M denotes Mach's number. Substituting $M = 0$ one obtains from Eq. (14)

$$\frac{q}{\tau} = \frac{\lambda_w}{\mu_w}\frac{T_0 - T_w}{U} = \frac{c_p(T_0 - T_w}{\rho_w U} \tag{15}$$

This is the relation known as Prandtl's or G. I. Taylor's formula, first discovered by O. Reynolds. Hence Eq. (14) gives the correction of this result for compressibility effects.

In the case $T_0 > T_w$, *i.e.*, when the wall is colder than the free stream, the effect of compressibility is to increase the heat transferred through the wall. However, it would be erroneous to interpret this result as an "improvement" in cooling because at high speed the heat produced in the boundary layer is of the same order as the heat transferred through the wall. In order to determine the efficiency of the cooling a complete heat balance must be made. For this purpose Eq. (14) does not give sufficient information and the velocity and the temperature distribution in the boundary layer must be computed. Such calculations were carried out for the particular assumption $T_w = T_0/4$, *i.e.*, for the particular case in which the absolute temperature of the wall is kept constant at a value equal to one-fourth of the temperature of the hot fluid. With the same assumption for the variation of μ as in Part I, the results shown in Fig. 2 and Fig. 4 were obtained. The variation of $C_f\sqrt{R}$ with M is similar to that obtained in the case without heat conduction through the wall. Also the highest temperature in the boundary layer is very high for extreme Mach numbers. However, the temperature maximum occurs some distance from the wall.

The heat transferred from the boundary layer to the wall can be calculated as follows:

The initial slope of the velocity profile is equal to

$$\left(\frac{\partial u}{\partial y}\right)_w = \frac{U}{L}\frac{\sqrt{R}}{\sqrt{n^*}}\left(\frac{\mu_0}{\mu_w}\right)\frac{\sqrt{R}C_f}{4} \tag{16}$$

By differentiation of Eq. (4) the relation between the velocity slope and the temperature gradient can be obtained. Using Eq. (7a) and substituting Eq. (16) into Eq. (5) then

$$(\partial T/\partial y)_w = K(T_0\sqrt{R}/4L\sqrt{n^*}) \tag{17}$$

where

$$K = (4^{0.76}/2)(0.75 + [(\kappa - 1)/2]M^2)\sqrt{R}\,C_f$$

Therefore, the heat transferred to a strip of unit width of the wall of length L per unit time is equal to

$$Q = \int_0^L \left(\lambda\frac{\partial T}{\partial y}\right)_w dx = \frac{K\lambda_w T_0\sqrt{R}}{2L}\int_0^L \frac{dx}{\sqrt{n^*}}$$

or approximately

$$Q \approx K\lambda_w T_0\sqrt{R} \tag{18}$$

where K is given in Table 2.

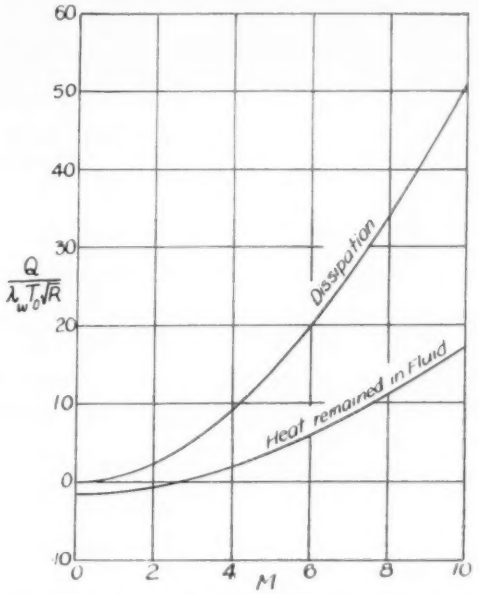

FIG. 5. Heat balance when the wall temperature is 1/4 of the free stream temperature.

TABLE 2

M	K
0	1.53
1	1.93
2	3.12
5	10.53
10	33.98

The total heat balance is shown in Fig. 5. The "dissipation" curve represents in dimensionless form the heat produced by friction per unit time and unit width of the plate. The lower curve shows the increase (or decrease) of the heat content per unit time and unit width. The difference of the ordinates corresponds to the heat transferred through the wall. It is seen that cooling takes place for $M < 2.6$. Beyond this limit more heat is produced by friction than the amount which can be transferred to the wall and, as a matter of fact, the fluid is heated.

In the case $T_w > T_0$, *i.e.*, when the wall is hotter than the free stream, the ratio between the heat transfer and the drag decreases with increasing Mach's number. This is shown in Fig. 6 where the ordinate represents the ratio between q/τ with compressibility effect (according to Eq. (14)) to q/τ without compressibility effect (according to Eq. (15)). The calculation was carried out for a gas temperature of $-55°$F. and a wall temperature of 180°F. and 300°F. It is seen that there is no cooling in the former case for $M = 1.69$ and in the latter case for $M = 2.08$. However, the decrease of cooling efficiency is appreciable even at much lower speeds. This emphasizes the benefit

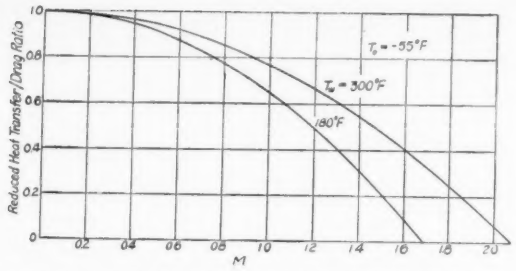

FIG. 6. The effect of high speed on cooling efficiency.

of the reduction of the speed of cooling air and the relatively poor efficiency of cooling surfaces exposed directly to a high-speed airstream. The curves in Fig. 6 being derived from Eq. (14) apply to laminar as well as to turbulent motion.

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Book Review

Metal Airplane Structures, by FLAVIUS E. LOUDY; The Norman W. Henley Publishing Company, New York, 1938; 455 pages, ill., $5.00.

Metal Aircraft Construction, by M. LANGLEY; Pitman Publishing Corporation, New York and London, 1937; 364 pages, ill., $4.50.

It is exceedingly interesting to set these books side by side. While one book is written in England and the other in the United States, both of them have the similar objective of presenting a general survey of metal airplane construction. Both books are excellent, comprehensive, and excellently illustrated. The main difference between them is that Mr. Langley has restricted himself mainly to illustration and discussion of various types of construction and shop processes, whereas Mr. Loudy presents less descriptive material, but discusses basic elements of structure without, however, going into detail stress analysis. Mr. Langley's book is to all intents and purposes an encyclopedia of metal construction with illustrations and descriptions of almost every well known metal machine built either in Europe or in the United States.

Mr. Loudy's book, first of all, includes a study of structural elements, such as tie rods, struts, channels, columns, composite sections, extruded shapes, etc. For all such sections data are given in regard to area, weight, etc. For struts various charts are given for various strength columns. Similar charts are given for characteristics of aluminum channels. Thus this first section of Mr. Loudy's book provides good reference to common structural elements. The chapter on Welded Joints presents information which is generally well known, but nevertheless is interesting to have in one volume. In discussing Riveted Joints, the author gives formulae for efficiency of riveted joints, takes care of eccentric calculations in loads, discusses bearing values, and presents some very interesting illustrations of riveted spars and of riveted joints. Special rivets such as the Thompson tubular rivet and the Iseman rivet receive attention. In dealing with Stressed Skin Design, Mr. Loudy gives a variety of charts showing maximum loads for plates and in general discusses the basic principles of stressed skin design in useful fashion. Here again data are collected from a variety of sources and useful references are provided. The same remarks apply to the treatment given to corrugated aluminum sheets. Due attention is given to tension field beams and the pertinent formulae are nicely collected for reference. Here again the principles are set forth in clear and concise fashion. Thereafter follows the review of the strength tests on thin wall duraluminum cylinders. From Chapter 7 and onwards, the material becomes descriptive. Various types of wings are briefly discussed, a classification of metal wing beams and spars is made, tests of torsional characteristics, etc., are presented. The chapter on Fuselage Design is also mainly descriptive with summaries of tests of fuselage deflection reported. A fairly complete bibliography is appended.

Mr. Loudy's book adds very little to the present knowledge of the subject nor is it a treatise on stress analysis. Nevertheless, it will be decidedly useful for reference, and to provide the designer or constructor or student with a general survey of metal construction.

On the other hand Mr. Langley's book makes no attempt at structural analysis nor at the presentation of test data. The strength of Mr. Langley's book lies in the large number of diagrams and illustrations throughout the book. Also, construction aspects are given due weight. For example in describing a Gloster rib, full details are given for its shop handling and almost everywhere in the book description of a design is followed by at least a hint of the shop methods employed. The book confirms the fact that British practice makes a far greater use of strip steel, but is in other respects closely analogous to American construction.

Mr. Langley also includes a chapter on flying boat hulls and seaplane floats—wood and metal. A chapter on Subsidiary Structures deals with fins, retractable engine mounts, retractable chassis, strut ends, control columns, tank structures, etc. Work shop processes and detail design are always interesting. Practice here is decidedly different from American practice, but may serve as an inspiration. Remarks on the spinning of fairing rings are instructive. Various processes of riveting are described. The appended tables and specifications relate of course to British practice.

Both books evidently deserve a place in the library of the airplane designer or constructor.

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Special Maps for Use with Loop Direction Finders

JAMES M. COBURN, JR., *New York University*

Presented at the Instruments Session, Sixth Annual Meeting, I. Ae. S.
January 24, 1938

THE radio range system with which the airways of the United States are equipped, while classed as an aid to navigation, in reality have not been so used to any great extent in the exact sense of the word "Navigation." Rather they have been used as an aid to contact flying in that the airplane proceeds from landmark to landmark just as it does in contact flying. In either case the exact position of the airplane and the ground speed is secured only by awaiting passage over a known landmark, the only difference being that in the case of "radio contact flying" the landmark is aural rather than visual. Position at other times, when following a defined airway, may be determined with satisfactory accuracy by the use of dead reckoning.

The addition of rotatable direction finding loops to the equipment of the airplane, whether they be of the so-called radio compass type or of the older and more simple aural direction finding loop, offers possibilities of a technique more nearly approaching the true sense of the word "Navigation" in that they provide a means of determining position at any time, whether on or off an established airway, without waiting for the moment of passage over a known landmark.

However, great care must be exercised in the introduction of new methods to the navigational technique of flight personnel. The complexities of operation of the modern airliner, coupled with the exacting nature of airways traffic control regulations, place a burden upon flight personnel which permit of little addition. Of a consequence, any change in routine must introduce a minimum of complications if it is to receive whole hearted acceptance. Unwelcome reception by flight personnel may easily cause resistance to the acquiring of the necessary familiarity with the equipment and technique, which may be acquired only in good weather, to the end that confidence in results secured when really needed might be lacking.

Of primary importance, then, in the introduction of navigational technique, is the development of equipments and methods which will impose a minimum of additional complications, either mental or mechanical, upon flight personnel, all operations to be performed by existing personnel without leaving their seats or diverting their attention more than momentarily from their other duties.

With these objectives in mind, before investigating the possibilities of equipment and methods which will meet them, it might be well to survey the accepted method of securing a radio position, or fix, using a standard direction finder of either radio compass or aural type and the well known Department of Commerce Aeronautical Sectional Charts. This is the method which has become standardized from many years use at sea for securing positions from both radio and visual bearings.

The successive steps are as follows:

(1) Tune the loop receiver to the frequency of the station upon which the bearing is desired and wait for positive identification. It may be necessary to rotate the loop to a position where maximum signal strength is secured.

(2) Rotate the loop to secure a null indication in the ear phones, or zero pointer reading on the left-right indicator.

(3) The angular indication of the pointer of the bearing indicator dial is read to secure the apparent relative bearing. The relative bearing is the angular difference between the fore and aft axis of the plane and a line joining the position of the plane and the radio station.

(4) From the calibration curve of the loop installation determine the corrected relative bearing corresponding to the apparent bearing.

(5) Read the magnetic compass to determine the compass heading of the airplane or the compass bearing of the fore and aft axis of the plane.

(6) Convert compass heading to magnetic heading by adding easterly deviation or subtracting westerly. Compass correction card must be consulted for these values.

(7) Convert magnetic heading to true heading by adding easterly variation or subtracting westerly. To determine the direction and amount of the variation, consult the isogonic lines on the sectional chart at the estimated position of the plane.

(8) The corrected relative bearing is added to the true heading of the airplane to secure the true bearing of the radio station from the plane.

(9) Add or subtract 180° to or from the true bearing to secure the reciprocal true bearing or the true bearing of the plane from the radio station.

(10) Establish true north through the radio station on a map including the radio station and the estimated position of the plane.

(11) With a protractor, the station in the center and aligned on the true north, lay off the reciprocal bearing.

(12) With a straight edge, draw line of position on the reciprocal bearing through the station.

(13 to 24) Repeat using a second station. The intersection of the two lines of position is the approximate position of the plane.

(25 to 36) Repeat using a third station if in any doubt about results obtained from two.

Disadvantages of Marine Methods Applied to Airliners

The difficulties of securing accurate results by following the foregoing procedure in flight may be easily imagined. Even among seafarers of many years experience cases continually arise where deviation and variation are applied in the wrong way, by addition where there should have been subtraction and *vice versa*. With many steps to the routine, one or more of them may be forgotten easily, particularly in times of mental strain and it is in these times that accurate results are most needed.

The disadvantage which precludes the possibility of the every day use of this method of position finding, however, comes to light when it is attempted to plot reciprocal bearings on the Aeronautical Sectional Charts within the confines of the cockpit. To do accurate plotting on any map requires that it be spread out upon a flat surface. In the marine service a chart table is provided for this purpose. Aeronautical Sectional Charts are of such a size that to spread out one of them would require a chart table of a size that is impossible in the already overcrowded cockpit. When it is considered that two or more of these maps might have to be joined together to cover the area that normally could be expected to be required in practice, the impossibility of the situation becomes evident.

Methods of Simplification

Simplification which will eliminate one step may be accomplished by incorporating the calibration of the loop in the bearing indicator dial. This means that a special dial must be made with a scale distorted so that the reading secured will be the corrected relative bearing (see Fig. 1). If a fleet of airplanes is to be equipped with direction finding loops mounted in exactly the same place and with identical receivers, calibration may be made in flight with one plane to secure the data for the constructing of the special dials with which to equip the whole fleet. This provides a very worthwhile simplification.

Additional simplification which makes this method of securing positions of practical value to the transport pilot rests entirely with the maps to be used and it is believed that this is answered by maps which were devised by Peter H. Redpath, navigational expert, for this purpose (see Fig. 2).

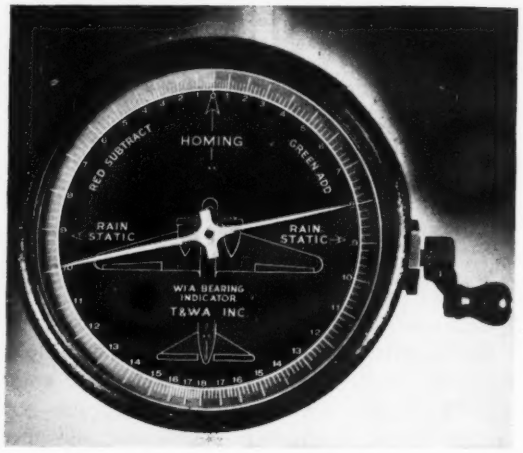

Fig. 1. Loop bearing indicator with distorted dial incorporating calibration. In this case relative bearings, instead of reading zero to 360°, read zero to 180° right or left.

A scale of 1:2,000,000 permits folding or cutting of a map into sections, easily handled by the pilot without leaving his seat. Sufficient area would be covered to include on one section the radio range stations on the airway to be used as well as those within useable range on adjacent airways on both sides. The size of this folded or cut sheet allows plotting without a chart table or special plotting board, the back of a navigation kit or a clip board for holding log sheets being all that is required.

The Lambert Conformal projection was chosen for the reason that a straight line on the map closely represents a great circle and it is furthermore the projection which is used in the Aeronautical Charts.

Cities and larger towns, with their names, rivers, lakes, and other visual landmarks were incorporated to eliminate the necessity for consulting other maps for this information. Elevations of the highest peaks and contours of elevations above normal cruising altitudes were included for warning purposes.

The location of radio range stations only, and not commercial broadcast stations is given since radio range stations identify themselves every few seconds whereas it may be necessary to wait as much as fifteen minutes for positive identification of a broadcast station. Consequently, under present conditions of infrequent identification, commercial broadcast stations are of little value.

A feature which greatly simplified the mechanics of plotting, by eliminating the necessity of using a protractor, was the printing of a compass rose around each radio range station. Since this feature also made unnecessary the establishing of the true North through the station, two steps of the previously accepted routine were at once eliminated.

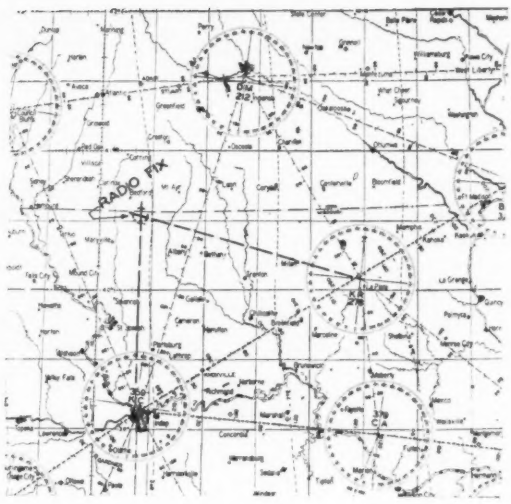

FIG. 2.

Another step was eliminated by orienting these compass roses on the magnetic North rather than the true North. With the roses thus oriented, the conversion of magnetic to true bearings is unnecessary. It may be argued that the use of the true North as a reference is necessary as the variation is continuously changing and normally will not, consequently, have the same value at the radio station and at the point from which the bearing is taken. This is quite true were it necessary or desirable to work to extreme accuracy. When accuracy is desired the correction for the difference in variation may be secured from the isogonic lines on the map, together with the correction for the error occasioned by the convergence of the meridians. However, with the abundance of radio range stations, the distance from any two workable stations that might be selected will probably be such that the correction is negligible. Nevertheless, the application of these corrections will be discussed later.

To eliminate another step, that of securing the reciprocal bearing, the magnetic compass rose, instead of having its zero degree position placed on the magnetic North was made a reciprocal rose by placing the 180° position to the magnetic North. Thus the time necessary to add or subtract 180° to or from the magnetic bearing is saved and the attendant possibility of error is removed.

With the simplifications introduced by these changes the routine of securing a radio fix appears very different, the steps being as follows:

(1) Tune the radio to the desired station.
(2) Rotate loop to secure a null indication.
(3) Read indication of the pointer on the bearing indicator.
(4) Add the relative bearing to the compass heading.
(5) Add or subtract the deviation.
(6) With a straight edge, draw line of position through the magnetic bearing on the reciprocal rose around the selected station.
(7) Repeat on second station.

The intersection of two lines of position is, of course, the approximate position of the airplane. In all but a few cases, where extreme accuracy is desired, or the approximate position is found to be several hundred miles distant from one or both of the stations, this approximate position will be sufficiently accurate for all practical purposes. However, from this approximate position it is very easy to work out a corrected position in cases where it is necessary or desired, as will be seen later.

Maps constructed along these lines were produced under the direction of Peter H. Redpath by Transcontinental & Western Air in the form of strips with the main TWA routes in the center. The scale, 1:2,000,000, is such that radio range stations on airways both North and South are included on a strip fourteen inches wide.

Instructions were issued to flight personnel outlining the exact procedure to be followed in obtaining a fix using these maps, as a supplement to a navigation course covering technical aspects of navigation by radio bearings in general and the various uses to which directional radio apparatus aboard a plane might be applied. Flight personnel were instructed to plot fixes at least once on every trip, checking the accuracy by visual landmarks where possible, to report upon the degree of accuracy and the time required to secure a fix by this method.

REPORTED RESULTS

The reports of results obtained were most satisfactory in nearly all cases. Positions were determined with an error of from one and one-half to five miles within one minute when using nearby stations. Accuracy within these limits left little to be desired and it was at first thought that it would be unnecessary to burden flight personnel with rules for applying corrections for the difference in magnetic variation and for the convergence of the meridians. Conditions arose, however, which made it advisable to include rules for these corrections in the instructions to flight personnel.

With certain conditions of radio reception and again in certain localities, bearings were obtained upon distant stations with greater ease than upon stations which were comparatively nearby. In these cases errors were reported of as much as 25 miles. Checking the results showed that this error was caused almost entirely by failure to allow for the difference in magnetic variation between the position of the airplane and the station upon which bearings were taken and the plotting error caused by the convergence of the meridians on the Lambert Conformal projection.

Simple rules were, therefore, worked out which could easily be memorized or carried in convenient memorandum form. These rules are:

(1) When the approximate position of the airplane is West of the station, add six tenths of one degree for each degree difference in longitude and also add one degree for each degree difference in variation.

(2) When the approximate position of the airplane is East of the station, subtract six tenths of one degree for each degree difference in longitude and also subtract one degree for each degree difference in variation.

If, then, an airplane desires a routine position check from distant stations the foregoing rules are applied to the dead reckoning position, but if radio reception has been lacking for some time it may be a saving in time to plot the approximate position, determine the amount of the correction to apply from the relation of this approximate position to the position of the stations upon which bearings were taken and then replot the actual position.

As an example of the use of these maps, assume that an airplane is flying on a magnetic heading of 78° and is not following the beam course of an airway. The position of the airplane is not known and it is desired to secure a radio fix.

Kansas City is tuned in at good volume so is selected as one of the triangulation points since the volume level indicates that it is not far distant. The loop is rotated for a null indication and the relative bearing is found to be 94°.

Kirksville, Missouri, is next tuned in and the relative bearing is found to be 22°.

Reducing these relative bearings to magnetic gives 78° plus 94° or 172° on Kansas City and 78° plus 22° or 100° on Kirksville.

Plotting these results, by use of the magnetic roses on the map, locates the airplane six miles southeast of Bedford, Iowa. (See Fig. 2.)

After it was definitely determined that a satisfactory degree of accuracy could be obtained, recommendations were made to the Bureau of Air Commerce that maps of the entire country, constructed in this manner, be prepared so as to be available for use by pilots of planes equipped with directional radio apparatus. This program is now under way and Aeronautical Chart for Radio Direction Finding No. 24 D.F. is now available.

Additional Requirements Contributing to Accurate Results

Up to this point, equipment and methods only have been studied and no attention has been paid to the human element. Before short cuts may be safely employed a thorough understanding of the underlying principles is necessary. Education, therefore, is essential in the theory of navigation by cross bearings, in the manipulation of the direction finding equipment itself, the taking of bearings, the solution of orientation problems and the many other uses to which directional radio apparatus may be put.

The mere equipment of an airplane with a directional radio device and the proper maps in itself is no guarantee of additional safety. Rather a false sense of security may be engendered unless steps are taken to assure that flight personnel acquire this familiarity with the equipment and methods.

As an instance, if the radio compass or aural direction finding loop is used as a simple homing device for following an airway in the manner which is generally advocated by the manufacturers of certain devices of this nature, there is an even chance that the plane will drift to the wrong side of the airway with attendant possibility of collision with traffic from the opposite direction. Manufacturer's pamphlets, demonstrating in a diagrammatic manner the curved flight path resulting from homing in a cross wind, always show the drift as being to the right of the center line of the airway. The fact that traffic in the reverse direction would be drifting to the left of the center line of the airway and in the way of traffic from the opposite direction, is strangely unnoticed.

This course of education should be standardized for all users of this equipment and they should be carefully examined to determine their ability to navigate with this equipment alone, under all conditions that may arise, lest they become a distinct menace to themselves and other traffic using the airways.

Accuracy Requirements for Auxiliary Equipment

Just as important is the assurance that reliance may be placed on auxiliary equipment upon which the accuracy of radio bearings depends. Since it is primarily necessary to know the angular relationship of the longitudinal axis of the plane to the magnetic North, which can be secured from the magnetic compass only, and the relative bearing of the radio station to the longitudinal axis, it is a requisite that the compensation of the magnetic compass and the calibration of the loop installation be performed by thoroughly competent men who will certify them to be correct.

With equipment and training such as has been outlined, it has been found that position finding is not an operation steeped in mystery, to be attempted only in cases of dire necessity, but rather one that is welcomed by flight personnel as a navigational method to be regularly employed in daily operations and one which rapidly becomes indispensable.

Obviously, if the addition of these special direction-finding maps coupled with simple training and education in the use of the combined equipment is all that is necessary to secure such results, these maps should be included, as fast as issued, in the equipment of all pilots using directional radio equipment and steps should be taken to assure a standard degree of proficiency in directional radio navigation.

On the Stability Problem in Hydrodynamics, II

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(Received February 23, 1938)

ABSTRACT

IT IS shown that the parabolic flow of a viscous fluid between two fixed walls is stable for small disturbances which have symmetrical streamlines.

(1) The striking phenomenon of the breakdown of the laminar flow of a viscous fluid has so far received a theoretical explanation only in the case of fluid which is confined between rotating cylinders.[1] The instability which is found there is also known to exist for an inviscid fluid and the effect of viscosity appears to be to defer the breakdown until some critical value of Reynolds Number. The parabolic laminar flow between fixed walls, which is the subject of the present discussion, was shown by Rayleigh to be stable in the limit of vanishing viscosity and small disturbances. It would thus appear that the instability which is observed is due to viscosity, a result which does not seem plausible. In fact, it is shown in this paper that in the particular case of symmetrical disturbances, stability persists for all Reynolds Numbers. The unsymmetrical disturbances have been discussed in a previous paper[2] for a limited range of small Reynolds Numbers, where it was shown that their damping is everywhere greater than for the symmetrical disturbances. It is thus not likely that the unsymmetrical disturbances will be unstable where the symmetrical disturbances are stable, although an explicit proof to that effect does not exist at present. If the above presumption is true, it must be concluded that the method of small oscillations is incapable of explaining the breakdown of the parabolic flow, the explanation for which must be looked for in the deviations of the actual conditions from those assumed in the theory. Possibly the breakdown can be brought about only by disturbances of finite instead of infinitesimally small amplitudes, as would seem to be indicated by the fact that in the ensuing turbulent flow the quadratic terms in the equations of motion play a significant role. Another possibility, which has been considered, is the inexistence of the parabolic flow at the intake and the possible instability of the boundary layer which is formed there. These factors involve even greater mathematical difficulties than the method of small oscillations and it would seem that their analysis will not be forthcoming until there is a satisfactory theory of turbulence.

(2) Let the motion take place in the x-direction and be bounded by the planes $y = \pm h$. If the half-width of the channel h is taken as the unit of length, the undisturbed laminar velocity U_0, which is assumed to be uniform along the x-direction is given by

$$U_0 = 1 - y^2 \tag{1}$$

The unit of velocity is equal to the maximum velocity at the center. By the method of small oscillations it is assumed that a perturbing velocity-field is superimposed on the above, which is of sufficiently small magnitude so that the squares of the velocities are negligibly small in comparison with their first powers. The above limitation is made not for physical reasons but rather for mathematical convenience, for otherwise the equations become non-linear and therefore untractable. Since the fluid is incompressible the velocities can be derived from a stream function

$$u = \partial\psi/\partial y, v = -\partial\psi/\partial x \tag{2}$$

where ψ is assumed to have the form

$$\psi = f(y) \exp. i\alpha(Ct - x) \tag{3}$$

ψ is thus periodic in x of wavelength $\lambda = 2\pi/\alpha$. C is in general complex $= a + ib$, where a is the phase velocity and the motion is damped if b is positive, or unstable if b is negative.

It is found[2] that $f(y)$ satisfies the equation

$$f^{IV} - 2\alpha^2\ddot{f} + \alpha^4 f + i\alpha R[2\ddot{f} + (1-y^2-C)(\ddot{f} - \alpha^2 f)] = 0, \tag{4}$$

where the dots stand for differentiation with respect to y and $R = Uh/\nu$, U being the absolute magnitude of the laminar velocity at the center and ν the kinematic viscosity. The condition of the vanishing of u and v at the walls implies that

$$f(1) = f(-1) = \dot{f}(1) = \dot{f}(-1) = 0 \tag{5}$$

Eq. (5) is satisfied only for particular values of C and the solutions of the system of Eqs. (4) and (5) for these values of C represent the possible oscillations of the parabolic flow which are periodic in x. From Eqs. (4) and (5) it is clear that $f(y)$ is either even or odd, so that the perturbations have either symmetrical or antisymmetrical streamlines. The stability problem reduces then, in the above, to the finding of the proper-values C and especially to the determination of the signs of their imaginary parts b. If the C's are all found to be positive the motion is proved to be stable for small disturbances, and unstable otherwise.

It was shown in I that the b's are all positive and the

TABLE 1

Values of $\alpha RC = \alpha Ra + i\alpha Rb$ from the Third Approximation

αR	10		100		1000		10000		100000	
α	αRa	αRb	αRa	αRb	αRa	αRb	αRa	αRb	αRa	αRb
0	3.52	9.48	2.16	21.65	0.21	21.60	0.021	21.60	0.0021	21.6
0	4.74	27.74	54.63	19.98	555.48	20.40	5555.55	20.40	55555.6	20.40
0	6.18	34.78	87.65	30.36	888.75	30.00	8888.87	30.00	88888.8	30.00
0.5	3.86	9.16	9.61	21.48	71.90	21.40	716.69	21.40	7167	21.40
0.5	4.94	27.41	55.11	19.56	562.63	20.03	5627.2	20.02	56273	20.02
0.5	6.44	35.04	87.71	30.57	889.77	30.18	8899.2	30.19	88990.7	30.19
1.0	4.71	8.68	24.84	22.17	216.35	21.89	2160.4	21.89	21589.6	21.89
1.0	5.37	26.97	56.80	18.01	585.99	18.80	5861.3	18.89	58613.1	18.81
1.0	6.88	35.73	87.87	31.20	892.78	30.69	8929.41	30.60	89308.6	30.68
0.4π	5.20	8.60	31.92	23.44	283.90	22.84	2835.5	22.84	28346.1	22.78
0.4π	5.57	26.96	58.51	16.67	605.31	17.88	6054.6	17.90	60522	17.93
0.4π	7.07	36.95	87.99	31.68	895.05	31.06	8952.4	31.05	89557.7	31.08
10	8.61	103	56.14	119.40	538.79	119.3	5385.12	119.30	53850.2	119.31
10	6.83	119	91.36	106.77	872.85	117.7	8715.83	118.08	87159.3	118.07
10	8.49	134	91.79	129.20	981.36	118.4	9829	118.0	98290	118.0
100	8.15	10018	55.5	10018	555	10018	5553.6	10018	55536	10018
100	8.15	10018	91.4	10018	889.8	10018	8886.9	10018	88869 6	10018
100	8.15	10018	97.5	10018	999	10018	9998	10018	99979	10018

motion therefore stable, for both the symmetrical and antisymmetrical types of disturbances for values of αR less than about 100. Similar results were obtained by a different method for the symmetrical disturbances by S. Goldstein.[3] It was further proved that stability exists for values of αR which are less than the smaller of the quantities $2\alpha^2$ and α^4. An inspection of the above theorem will show that it can be amplified to prove stability for values of αR which are *equal to* the smaller of the quantities $2\alpha^2$ and α^4. Thus, by an application of the above theorem stability can be ascertained for the range of values of α and αR which are to the left of the thick lines in Tables 1 and 2.

It shall now be shown that for very long wavelengths such that $\alpha \longrightarrow 0$, but αR remains finite, the parabolic flow is stable for symmetrical disturbances. Eq. (4) reduces then to

$$f^{IV} + i\alpha R[2f + (1-y^2-C)\ddot{f}] = 0 \qquad (6)$$

which integrates to

$$\dddot{f} + i\alpha R[2yf + (1 - y^2 - C)\dot{f}] = A \qquad (7)$$

If f is even, the terms on the left of Eq. (7) are odd and therefore $A = 0$. One solution of Eq. (7) is

$$f_1 = (1-y^2-C) \qquad (8)$$

If f_2 is the second even solution of Eq. (7) one shall have a general even solution of the form $Af_1 + Bf_2$. The boundary conditions, Eq. (5), imply that at $y = 1$

$$f_1\dot{f}_2 - \dot{f}_1 f_2 = 0 \qquad (9)$$

But the left-hand side of Eq. (9) equals the value of the quantity in brackets in Eq. (7) at $y = 1$, if in the latter f_2 is written for f. Hence it follows that

$$\dddot{f}_2(1) = 0 \qquad (10)$$

Now multiply Eq. (6) by $\ddot{\bar{f}}$, where $\bar{f}$ is the complex conjugate of f, and integrate from $y = -1$ to $y = +1$. From the identities

$$f^{IV}\ddot{\bar{f}} = \frac{d}{dy}(\dddot{f}\ddot{\bar{f}}) - \dddot{f}\dddot{\bar{f}}, \quad \dddot{f}\ddot{\bar{f}} = \frac{d}{dy}(\ddot{f}\dot{\bar{f}}) - \ddot{f}\dot{\bar{f}}$$

the boundary conditions and Eq. (10) one obtains

$$-\int_{-1}^{1}\dddot{f}\dddot{\bar{f}}\,dy + i\alpha R[-2\int_{-1}^{1}\dot{f}\dot{\bar{f}}\,dy + \int_{-1}^{1}(1 - y^2 - C)\times \ddot{f}\ddot{\bar{f}}\,dy] = 0, \quad (11)$$

which shows that the imaginary value of C is positive for all values of αR. The range of stability has thus been extended to the values at the tops of Tables 1 and 2.

(3) Methods of extending the stability investigations to larger Reynolds Numbers were suggested in I, but these proved to be laborious and the more so the larger the values of R. It appeared, therefore, that a numerical method whose applicability is not limited to any definite range of values of R would be

TABLE 2

Values of $\alpha RC = \alpha Ra + i\alpha Rb$ from the Fourth Approximation

αR	10		100		1000		10000		100000	
α	αRa	αRb	αRa	αRb	αRa	αRb	αRa	αRb	αRa	αRb
0	3.80	9.67	11.63	37.92	0.956	36.58	0.0955	36.57	0.00956	36.57
0	5.42	32.02	38.58	32.13	437.23	33.82	4374.97	33.83	43749.99	33.83
0	5.63	55.40	72.18	32.84	750.07	36.20	7500.01	36.27	75000	36.27
0	6.41	62.91	90.1	57.1	936.75	53.40	9374.92	53.33	93750	53.33
0.5	4.15	9.38	20.37	41.18	68.96	37.37	678.93	37.36	6788.24	37.37
0.5	5.60	31.87	37.59	28.15	444.77	32.27	4451.30	32.29	44513.30	32.27
0.5	5.62	55.04	72.25	32.96	752.29	36.18	7522.36	36.24	75223.56	36.24
0.5	6.66	63.10	90.1	57.1	937.05	53.58	9378.17	53.51	93782.48	53.51
1.0	4.74	8.99	29.26	48.45	200.82	41.52	1995.67	41.46	19955.48	41.46
1.0	4.57	31.61	45.33	18.91	472.39	27.26	4728.18	27.33	47282.25	27.34
1.0	7.32	54.57	72.38	33.31	759.20	35.98	7591.99	36.04	75919.85	36.04
1.0	7.07	63.71	90.06	58.20	937.96	54.11	9387.83	54.03	93879.10	54.02
0.4π	5.40	8.89	32.97	50.83	256.90	44.92	2557.60	44.83	25574.69	39.71
0.4π	6.28	31.55	50.37	16.92	497.49	24.14	4977.06	24.25	49770.99	29.38
0.4π	5.65	54.62	72.38	33.31	764.76	35.69	7648.13	35.76	76481.37	35.76
0.4π	7.25	64.20	90.06	58.2	938.65	54.51	9395.20	54.41	93952.78	54.41
10	8.64	102.86	51.05	137.3	422.90	135.77	4221.16	135.75	42210.76	135.74
10	6.90	121.44	71.76	107.1	734.98	131.80	7343.79	131.86	73437.30	131.86
10	6.63	145.89	91.35	130.6	975.32	134.65	9844.20	133.95	98449.97	133.95
10	8.45	161.75	92.09	156.9	929.33	129.72	9216.16	130.38	92155.02	130.38
100	7.81	10032	59.88	10025	426.95	10033	4272.85	10009	42728.5	10009
100	7.81	10032	78.11	10032	781.05	10032	7810.5	10032	78105	10032
100	7.81	10032	81.54	10053	987.20	10045	9868.68	10069	98686.8	10069
100	7.81	10032	92.9	10018	929	10018	9290	10018	92900	10018

preferable. Such is the method of central differences which was used successfully by Jeffreys[4] in a problem of thermal stability. It consists in subdividing the range of integration into a finite number of equal intervals and using the finite differences for the differentials in Eq. (6). Let the interval from -1 to $+1$ be divided into $2n$ equal parts so that the length of one interval d equal $1/n$. Denote the values of f at some given point by Z_0, and at the ends of the neighboring intervals by the scheme

$$Z_{-3},\ Z_{-2},\ Z_{-1},\ Z_0,\ Z_1,\ Z_2,\ Z_3$$

One then has for the first, second, and fourth differentials of f the expressions

$$\dot{Z} = \frac{1}{2d}(Z_1 - Z_{-1}),\ \ddot{Z} = \frac{1}{d^2}(Z_1 - 2Z_0 + Z_{-1}),$$

$$Z^{\mathrm{IV}} = \frac{1}{d^4}(Z_2 - 4Z_1 + 6Z_0 - 4Z_{-1} + Z_{-2}) \quad (12)$$

The procedure will be illustrated for the case $n = 2$, $d = 1/2$. Denote the values of f at $y = 0$, and $1/2$, respectively, by a and b. The value of f at $y = 1$ must be zero by Eq. (5), and in order to make $\dot{f}(1)$ vanish, a value b is assigned to $f(3/2)$ even though $y = 3/2$ is outside the range of integration. Since an even solution is being considered, one has to assign for f at $y = -1/2, -1, -3/2$, the values b, 0 and b. Then at $y = 0$

$$f = a,\ \ddot{f} = 4(2b - 2a),\ f^{\mathrm{IV}} = 16(-8b + 6a)$$

and at $y = 1/2$

$$f = b,\ \ddot{f} = 4(a - 2b),\ f^{\mathrm{IV}} = 16(8b - 4a).$$

It follows now from Eq. (4) that

$$16(-8b + 6a) - 8\alpha^2(2b - 2a) + \alpha^4 a + i\alpha R[2a + (1 - C)(8b - 8a - \alpha^2 a)] = 0, \quad (13)$$

$$16(8b - 4a) - 8\alpha^2(a - 2b) + \alpha^4 b + i\alpha R[2b + (3/4 - C)(4a - 8b - \alpha^2 b)] = 0. \quad (14)$$

In order that Eqs. (13) and (14) be consistent, the determinant of the coefficients of a and b must be made to vanish.

Hence:

$$\{96 + 16\alpha^2 + \alpha^4 + i\alpha R[2 - (1 - C)(8 + \alpha^2)]\} \times \{128 + 16\alpha^2 + \alpha^4 + i\alpha R[2 - (3/4 - C)(8 + \alpha^2)]\} = \{-128 - 16\alpha^2 + i\alpha R \times 8(1 - C)\} \times \{-64 - 8\alpha^2 + i\alpha R(3 - 4C)\}. \quad (15)$$

This is a second order equation in C which upon development and denoting αRC by x becomes

$$x^2(32 + 16\alpha^2 + \alpha^4) - x[\alpha R(24 + 24\alpha^2 + 1.75\alpha^4) + i(768 + 352\alpha^2 + 48\alpha^4 + 2\alpha^6)] - (4096 + 1536\alpha^2 + 352\alpha^4 + 32\alpha^6 + \alpha^8) + \alpha^2R^2(8.5\alpha^2 + 0.75\alpha^4) + i\alpha R(256 + 248\alpha^2 + 38\alpha^4 + 1.75\alpha^6) = 0. \quad (16)$$

The two roots of Eq. (16) which are complex and are functions of α and αR correspond, respectively, to the first mode of oscillation with no nodal lines and to the second mode of oscillation having one nodal line. These roots turn out to have positive imaginary parts for all values of the arguments α and αR. One can in fact dispense with the explicit solution for the roots and determine the sign of their imaginary parts directly from Eq. (16) by a well known algebraic operation. But the latter procedure, although offering a great time-saving in the subsequent approximations, does not reveal the degree of convergence of the successive approximations.

Clearly if intervals of $^1/_3$ and $^1/_4$ are next used, one shall obtain instead of Eq. (16) equations of the third and fourth degree, respectively. The solution of these latter equations, which turned out to be extremely laborious, is given in Tables 1 and 2. The degree of convergence of the solutions for the range of values of α and αR which are to the right of the thick lines are of primary interest. There, the first roots, for which the convergence should be best, differ in absolute magnitude by less than 10 percent. The imaginary values differ considerably, due to the fact that the real parts are so much larger and consequently a small error in the latter involves a larger error in the former. The first roots in the column for $\alpha R = 10$ are in close agreement with the values obtained from the formulae in I. For $\alpha = 100$ the imaginary parts are approximated to less than 1 percent already in first approximation, where $d = 1$. It did not seem worthwhile to proceed with a fifth approximation, especially since the evidence as to the sign of the imaginary parts is sufficiently convincing.

(4) It is of interest to compare the oscillations of a viscous fluid with those of an inviscid fluid. This can be done in the limiting case of long wavelengths. The oscillations of an inviscid fluid are then governed by the equation

$$2f + (1 - y^2 - C)\ddot{f} = 0 \quad (17)$$

subject to the condition that $f(1) = 0$. The solution of Eq. (17) being $(1 - y^2 - C)$, it follows that $C = 0$. In the case of a viscous fluid it can be shown that, if α vanishes in such a way that $\alpha R \longrightarrow 0$ as $R \longrightarrow \infty$, the oscillations are damped as if the fluid were still. C then vanishes for all modes of oscillation like (α/R), thus approaching in the limit the value for an inviscid fluid. When α vanishes in such a way that $\alpha R \longrightarrow \infty$ as $R \longrightarrow \infty$ the values of the first roots at the tops of Tables 1 and 2 show a similar behavior. The other roots seem to remain finite, but the real parts are continuously decreasing with the successive approximations, so that it is not impossible, though unlikely, that they too approach zero.

If absolute units of length and time are reverted to, ψ is of the form

$$f(y) \text{ exp. } i\alpha\left(\frac{CUt}{h} - \frac{x}{h}\right)$$

Now it seems from Tables 1 and 2 that with the exception of the first root at $\alpha = 0$, C approaches with increasing R a value $(a + ib/R)$.
Hence:

$$\frac{CUt}{h} - \frac{x}{h} \longrightarrow \frac{aUt}{h} + \frac{ibt\nu}{h^2} - \frac{x}{h} \quad (18)$$

Now consider a disturbance of wavelength $2\pi/\alpha$ which is generated at the intake. According to Eq. (18) the time T it will take this wave to cross a given length L of the channel is of the order of L/aU. During that time its amplitude will have decreased to exp. $(-ab\nu L/aU)$ = exp. $(-\alpha bL/hR)$. Hence, as the Reynolds Number is increased the fraction by which the amplitude of a given small disturbance will be damped after crossing a given distance will decrease continuously. Presumably this tendency for increasing persistence with increasing R may turn to instability as the amplitude of the disturbance is increased.

It would be of interest to apply the method of central differences to velocity profiles with inflection points, which from the work of Tollmien[5] are known to be unstable for an inviscid fluid. It should be expected that viscosity will affect stability for small Reynolds Numbers and that beyond a certain critical Reynolds Number the flow will become unstable.

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On the Triple Origin of Air Forces*

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AIR forces have their origin partly in the shape of an obstacle obstructing the free motion of air, and partly in the condition at its surface; both causes generally contributing together at the same time. A third and important item is the rate of change of these conditions giving rise to the creation of the air forces, so that the acceleration or change of flow pattern must be taken separately into account.

That is well known, but not enough attention has sometimes been given to the fact that these airforce portions or components are of different dignity. The shape forces are of higher dignity and may be called noble, the surface forces are of less dignity and must be considered as base. This distinction is even more pronounced under accelerated conditions. Noble and base are to be taken in the sense used in old times by the alchemists and still used by the modern chemists with respect to metals; gold being designated as a noble metal, and lead as a base metal. Modern chemistry also speaks of noble gases. Nobility and baseness are traits of human character reflected through all nature, which essential unity between humanity and the rest of nature is sometimes lost sight of in modern times. Formerly, people realized more clearly that inanimate nature depicts and manifests the traits of human character. Be that as it may, there is no doubt but that the aerodynamic shape forces are much more positive, simple, predictable and require less correction, in comparison with idealized theory, than surface forces. These can only be conditionally predicted, and require correction, and a less uniform and certain one, in applying theory. The acceleration shape forces in particular are almost certain to make their appearance right away, nothing short of an actual compression of the air can retard them. The acceleration forces of surface effects, in contrast thereto, can hardly be predicted at all, and as far as they can be, are slow to develop and uncertain to realize full strength.

The digest of the total air force, as measured in the laboratory or elsewhere by the designer, requires therefore as an intermediate step its analytical separation into noble and base constituents, the same to be united again for the final engineering use of the information. In that way, and in that way only, can one wind-tunnel reading be made to answer a thousand questions, and the wind-tunnel tests have to answer that many questions lest wind tunnelling degenerate into fruitless weighing and air blowing. The wind-tunnel model has to be dimensioned previous to the test, not afterward. The question, incarnated in the model, and put to the tunnel, has itself to be sensible, if a useful and sensible answer is to be expected. It is physically impossible to test all combinations imaginable. Very often, the design has to be based on past tests and yet combines features never combined before in any test. Therefore, let wind-tunnel results be available in a form to be used broadly, quickly, easily, and still reliably.

These general remarks may be illustrated by the discussion of certain aspects of airfoil forces. Even in two-dimensional flow, the wing forces are decidedly of mixed origin. The profile or friction drag is beyond doubt a surface effect, and therefore of decidedly base nature. It can hardly be predicted by pure reasoning. It fails conspicuously to follow the legitimate square law of Newton; it rather flirts and plays with the less legitimate law of Reynolds, without even showing particular faithfulness to that. The profile drag permits itself to be lured off the straight path by every secondary and accidental influence. The physical conditions of the airfoil surface, the interference of adjacent objects, local disturbances in the air motion, and other influences sway this base and unpredictable item.

With the pure lift of the two-dimensional flow, as contrasted to the lift of aerodynamic induction, things are already a little better. Kutta's principle of a smooth trailing edge flow is indeed a surface effect, but only a small portion of the surface, the trailing end, is involved. The forward portion of the airfoil acts by its shape rather than by its surface to bring about the particular condition at the trailing edge. The lift is therefore less base than the profile drag, although it has not yet attained to nobility of the highest order. It is at least conditionally possible to precompute the lift by pure reasoning, and the correction needed to compute the actual lift from the theoretical one is not extreme. The zero lift direction of a wing section is even of higher nobility, and there is practical agreement between theory and practice in that respect. This is so because the absence of lift is predicated on the absence of surface effects calling for lift. As with humans, so also with surface effects: nonfeasance, doing nothing, sitting

* In view of the author's preeminent place in the history of aeronautical science, his philosophical views, even though expressed in a somewhat unorthodox fashion, should be of general interest. *The Editor.*

still, can be more safely expected under conditions favorable thereto, than positive action.

Under simplest assumptions, the slope of the two-dimensional lift curve is predicted to be 2π. During the war days, wind-tunnel tests prompted the author to substitute *one tenth per degree* for this theoretical value, a rule which has recently been confirmed and could not be improved by the most painstaking tests. *One tenth* corresponds to less than 2π, so that there seems to be indicated a general lift deficiency of 10 to 11 percent. The theoretical lift fails to materialize at least by that much. Further progress of practical and useful aerodynamics seems to be tied up with looking a little closer into this question. Propeller blades seem to have a much steeper lift slope than ordinary airfoils. The actual deficiency of airfoils is often much larger than 11 percent, because the theoretical slope of 2π holds only for infinitely thin sections. With thick sections, the above factor has to be multiplied by the sum of chord and thickness, in first approximation, rather than by the chord only. Thus, wing sections 10 percent thick would have a theoretical lift slope of 2π times 1.1. Its actual lift deficiency is therefore near to 20 percent rather than 10 percent. Suppose now that the lift deficiency is caused chiefly by the thickness of the tail portion, or behind parts, of the wing section, this being the place of base surface effects, rather than by the overall thickness of the section. What would then be expected from a thin section having a thick tail, so that the theoretical lift increase caused by overall thickness is absent, but the lift deficiency caused by tail thickness is present? A lift slope smaller than the almost generally found *one tenth per degree*. This crocodile effect suggests itself easily to the mind weighing off baseness against nobility, in advance of any measuring. This very same thing has recently been verified experimentally The N.A.C.A. 0012–65 section is such a crocodile section with a small body and a thick tail. Its lift slope is as much as 16 percent smaller than the almost universally observed *one tenth*. Here then is a case where the secondary influences, which ordinarily do cancel, fail to cancel in consequence of unusual combinations. This shows then that it pays to consider separately those items which ordinarily cancel each other, to separate what is separable, and to unite again at the proper time. This is a case where separation enables the advanced research worker to answer a question about an unusual wing section from tests made with the usual wing sections.

Applying the same logic to the question of airfoil equilibrium, let it be remembered that the moment about the point of application of the circulation lift is particularly noble, being in its entirety a shape effect. This *central point* of pure lift application is, with thin wing sections, at about 50 percent chord station. With thicker ones the central point tends to shift back, say by one fourth of the radius of curvature of the leading edge under ordinary conditions. The theoretical moment about that central point is probably always 100 percent realized. The lift is not that fully realized, as just discussed. Now, the distance of the important *aerodynamic center* from the central point is determined by the ratio of the moment slope to the lift slope. There results 25 percent in the simplest theoretical case, and hence the first choice of the aerodynamic center is 50 percent minus 25 percent giving 25 percent. In a representative case, the radius of curvature of the edge may be 4 percent, shifting the central point to 51 percent. The moment is not deficient, but smaller than the one corresponding to zero thickness by $^1/_2$ times 0.1^2, giving $^1/_2$ percent. Hence the aerodynamic center should be ahead of 51 percent by 25 percent $\times$ 0.995/.9 or by 23.5 percent, giving about 23 percent. It actually is generally about at 24 percent. All these relations should be carefully checked and the truth established. They are not final.

Now suppose again that the airfoil has a crocodile section with a thick tail and slender body. The central point is then less displaced to the rear and closer to 50 percent, but the distance of the aerodynamic center therefrom is increased, because the lift slope is decreased. A crocodile section should therefore have an unusually forward position of its aerodynamic center. The wind-tunnel test with the N.A.C.A. 0012–65 section again confirms this reasoning. Its aerodynamic center seems to be at 22 percent, closer to the leading edge than usual.

The broad subject is far from being exhausted, but the space and time allotted to this paper is. These examples may be enough to illustrate the general principle. March divided and fight united is not only a good military rule of tactics, but it is also of highest value for obtaining fruits of scientific research which are really digestible by the engineer.

A Water Tank for Model Tests on the Motion of Airships in Gusts

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SUMMARY

A water tank and the auxiliary apparatus for determining the free motion of airships in gusts is described. The setup is in operation at the Guggenheim Airship Institute and a series of tests, financed by the Navy Bureau of Aeronautics, is being carried out. The method utilizes a water basin across a section of which a stream of water, simulating a gust, flows. An airship model immersed in the water and moved across the gust is free to turn and drift. The position and azimuth of the model are registered as functions of the time. The interpretation of the results in terms of forces aud moments acting on the ship are discussed.

AN AIRSHIP has a mean density equal to that of the air it displaces and so forms an integral part of the atmosphere in which it moves. As a result it may be expected to carry out a considerable and complicated movement under the action of large scale gusts, *i.e.*, wind disturbances covering a volume comparable in size to that of the airship. This motion must be taken into account before an accurate determination of the forces acting on the ship can be obtained.

This paper deals with a method for determining experimentally the motion and, from this motion, the forces and pitching (yawing) moments acting while the airship flies through vertical (crosswind) atmospheric gusts.

In order for the motion of a model through an artificial gust to duplicate on model scale the motion of a full scale airship through an atmospheric gust, the following conditions must be satisfied: (1) the model must have the same mean density as the fluid in which it moves, (2) the radius of gyration of the model about a transverse axis through its center of buoyancy must be l times that of the full scale airship, where l is the model scale, (3) the wave length or the width of the transition region from zero to full gust velocity of the artificial gust must be l times that of the atmospheric gust, and (4) the ratio of flight velocity to lateral velocity in the gust must be the same for the model as for the full scale case. These conditions can be satisfied for a wide variety of gust structures by the method described below.

As far as the author is aware this method represents the first experimental approach to the problem of determining gust forces on airships with a model which is free to turn and drift under the action of the gust. The investigation was suggested by the author and is being carried out at the Guggenheim Airship Institute with the financial support of the Navy Bureau of Aeronautics. This support and suggestions by members of the Bureau are gratefully acknowledged.

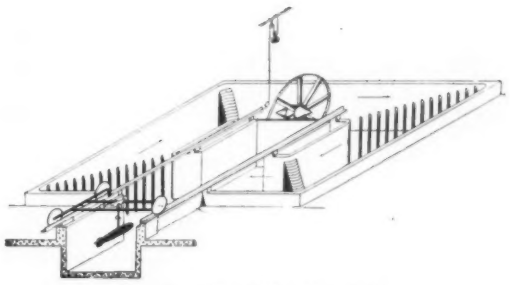

FIG. 1. Sketch of water tank.

The experiment is being carried out in water, since it would be extremely difficult to build a small scale model of an airship which would float in air and still be strong enough to withstand the gust forces. The method involves the use of a water basin (Fig. 1), across a section of which a lateral flow, simulating a gust, is maintained. The lateral flow is maintained in a closed channel intersecting the basin. The basin is 23 ft. long, eight ft. wide, and is filled with water to a depth of five ft. The channel in which the lateral flow is generated is also eight ft. wide and five ft. deep.

The airship model is suspended at its center of buoyancy from a carriage which is free to move laterally across the channel on tracks forming the cross members of a main carriage. The model is suspended at a depth of $2^1/_2$ ft. below the surface of the water. The main carriage and, with it, the model and cross carriage, are drawn along the basin in a direction perpendicular to the gust direction by means of weights acting over pulleys. The model is free to turn and drift under the action of the gust. The position of the center of buoyancy and the azimuth of the model are recorded by means of pens on a chart the movement of which is proportional to the movement of the model along the basin. Fig. 2 is a photograph of the setup.

The model used is a 1/150 scale model of the AKRON-MACON, 62.8 in. in length, with a maximum diameter of 10.6 in. It was cast in five parts of magnesium alloy with a wall thickness of about $^3/_8$ in. Weights were placed in the hollow space until the weight of the model plus cross carriage was equal to the weight of the water displaced by the model. The weights were distributed inside the model so that the radius of gyration about

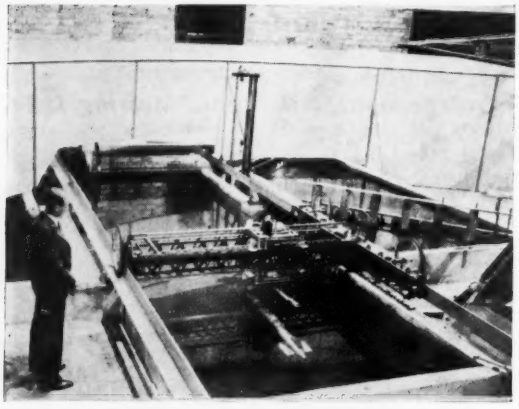

FIG. 2. Photograph of water tank showing carriage and airship model.

a transverse axis through the center of buoyancy was 1/150 that of the full scale airship. The moment of inertia of the AKRON-MACON was 73,000,000 kg. m. sec.^{2} at sea level and 67,000,000 kg. m. sec.^{2} at 3000 ft. Taking a mean value of 70,000,000 kg. m. sec.^{2} the moment of inertia of a 1/150 scale model which will float in water is 0.735 kg. m. sec.^{2} The moment of inertia of the model was determined by comparing the period of oscillation of the model suspended by a wire at its center of buoyancy with that of a bar of known moment of inertia suspended by the same wire.

Fig. 3 shows a gust velocity profile in a plane approximately along the model path. The nose of the model strikes the gust along the line A, two feet from the wall of the basin. The model leaves the gust approximately along the line C, two feet from the far wall of the basin. In the intervening region the velocity is quite constant as is shown by observations along the lines B, B_1, and B_2. For this velocity profile the stern of the model is well into the gust before the nose starts to leave. The passage of the model into the gust represents in this case the passage of the airship through a wind shift such as is encountered particularly at cold fronts or during thunderstorms.

The main gust velocity for the profile shown is 2.5 in. per sec. A model velocity of, say, 8.3 in. per sec. through the gust gives a ratio of gust velocity to model velocity of 0.3. The width of the transition region from zero to full gust velocity is about 7 in. The motion of the model under these conditions is, to scale, that of an airship flying through a windshift of 90°, the wind velocity component normal to the initial flight path increasing from zero to 0.3 times the flight velocity over a distance of 150 × 7 in. or 87 ft. For a flight velocity of 100 ft. per sec. the corresponding gust velocity would be 30 ft. per sec. According to existing data [1,2,3] a gust of this magnitude with a transition width as low as 87 ft. is near the upper limit of disturbances encountered in the free atmosphere.

The Reynolds Numbers attained in these tests are necessarily low. Model velocities range from 6 to 18

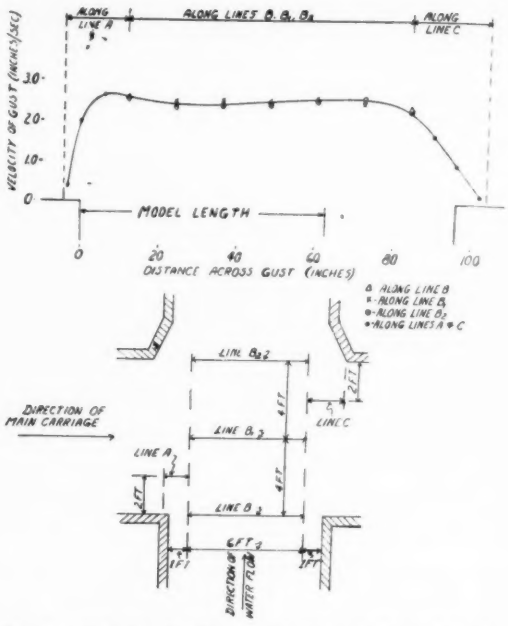

FIG. 3. Velocity distribution across gust channel along approximate path of model.

in. per sec., giving a range of Reynolds Numbers from 60,000 to 160,000.

The force in the gust direction and the moment of the forces about the center of buoyancy can be obtained from the cross and angular accelerations of the model, respectively. These accelerations can be found graphically from the records of position and angle as a function of the time.

One of the most important quantities derivable from these curves is the resultant quasi-steady angle of attack of the fins, particularly the maximum angle of attack attained for any particular value of the ratio of gust velocity to model velocity. This angle of attack with respect to the resultant flow follows directly from geometrical considerations taking into account the gust velocity and the motion of the model in determining the resultant flow.

The severity of a gust of a given structure is determined by the ratio of gust velocity to model velocity and by the width of the transition zone from zero to full gust velocity. In the set-up described both of these factors can be varied between wide limits. A variety of "gust structures" can also be duplicated and by varying the width of the gust the effect of gust wave length on the forces experienced by the airship can be determined.

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Determination of Profile Drag from Measurements in the Wake of a Body

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Introduction

RECENTLY the method of profile drag determination by measurements in the wake of a body has received widespread interest by the aerodynamicists. It seems pertinent therefore at this time to give a simple deduction of the final working equations and to show clearly what the basic assumptions are upon which these equations are founded.

Fundamental Principles

When a real, viscous fluid flows past a body, the portion of it which comes in contact with the body is slowed down by friction. Thus there is a velocity deficiency in the wake of the body. Outside the wake the motion is not much affected. The problem is now to find a relation between this velocity deficiency and the drag which the body experiences.

The total drag of a body is in general composed of two portions, the induced drag associated with a net lift acting on the body and the profile drag due to the skin friction and separation losses. An attempt shall be made to determine expressions for these two drag components separately, making use of (1) the equation of continuity, (2) the momentum principle, and (3) Bernoulli's equation along a streamline.

Consider a control surface enclosing the body consisting of a cylinder with plane faces at the front and rear. Finally, let the radius of the cylinder tend to infinity. Then the continuity equation states that the net inflow into this cylinder is zero, *i.e.*

$$\int_{S_0} \rho u_0 \cdot dS - \int_{S_r} \rho q_r \cdot dS - \int_{S_1} \rho u_1 \cdot dS = 0$$

where

S_0 plane face of cylindrical control surface ahead of body

S_1 plane face of cylindrical control surface behind body

S_r cylindrical surface

u_0, v_0, w_0 are x, y, z components of velocity at surface S_0

u_1, v_1, w_1 are x, y, z components of velocity at surface S_1

u_r, q_r are the velocity components along and normal to cylindrical surface, respectively

ρ is the air density

Since the surfaces S_0 and S_1 are equal in area, the equation of continuity reduces to

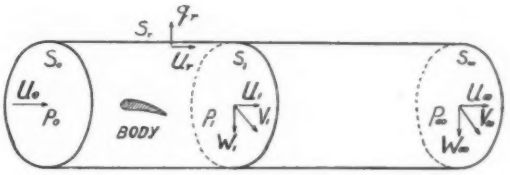

FIG. 1.

$$\int_{S_r} \rho q_r \cdot dS_r = \int_{S_1} \rho(u_0 - u_1) \cdot dS \qquad (1)$$

The momentum principle states that the sum of the x-components of the forces acting on the fluid enclosed between the control surface and the body is equal to the outflow of momentum in the x-direction. This gives

$$-\int_{S_0} \rho u_0^2 \cdot dS + \int_{S_r} \rho u_r q_r \cdot dS + \int_{S_1} \rho u_1^2 \cdot dS = \int_{S_0} p_0 \cdot dS + \int_{S_r} \tau \cdot dS - \int_{S_1} p_1 \cdot dS - D$$

where p_0 and p_1 are the pressures acting at surfaces S_0 and S_1, respectively, τ is the shear force acting on the cylindrical surface and D is the total drag acting on the body.

Since as $r \longrightarrow \infty$, the velocity along the cylinder $u_r \longrightarrow u_0$

$$\int_{S_r} \rho u_r q_r \cdot dS \longrightarrow u_0 \int_{S_r} \rho q_r \cdot dS = \int_{S_1} \rho u_0 (u_0 - u_1) \cdot dS$$

In general, there is no shear force τ acting on the cylindrical shell, and thus the equation reduces to

$$D = \int_{S_0} (p_0 + \rho u_0^2) \cdot dS - \int_{S_1} (p_1 + \rho u_1^2) \cdot dS - \int_{S_1} \rho u_0 (u_0 - u_1) \cdot dS \qquad (2)$$

or

$$D = \int_{S_1} (p_0 - p_1) \cdot dS + \int_{S_1} \rho u_1 (u_0 - u_1) \cdot dS \qquad (3)$$

Deduction of Expression for Induced Drag

Eqs. (1) to (3) hold for an ideal fluid as well as for a real fluid. However, it is known that in an ideal fluid the total drag which can exist must be induced drag. Thus denoting with subscripts i the quantities

referring to the ideal fluid, the following relations are found. The equation of continuity becomes

$$\int_{S_r} \rho q_{ri} \cdot dS = \int_{S_1} \rho(u_0 - u_{1i}) \cdot dS = 0 \qquad (1a)$$

This can be easily seen by considering the induced velocities due to a single horseshoe vortex and a doublet located within the control surface. Because of the symmetry there is no net influx or outflux over the infinite cylindrical boundary. The actual body can be made up by a superposition of doublets and horseshoe vortex pairs, and thus Eq. (1a) applies.

The momentum principle gives

$$D_i = \int_{S_0} (p_0 + \rho u_0^2) \cdot dS - \int_{S_1} (p_{1i} + \rho u_{1i}^2) \cdot dS \qquad (2a)$$

or

$$D_i = \int_{S_1} (p_0 - p_{1i}) \cdot dS + \int_{S_1} \rho u_{1i}(u_0 - u_{1i}) \cdot dS \qquad (3a)$$

The above integrals can be transformed by making use of the fact that Bernoulli's equation holds along a streamline even in vortex flow and thus the total head

$$g = p + (\rho/2)(u^2 + v^2 + w^2)$$

is the same along every streamline, namely, equal to that at S_0. Thus $g_0 = g_{1i}$. Up to this point the deductions are rigorously correct. Now, however, the assumption is introduced that the plane S_0 is far enough ahead of the body such that it experiences negligible induced velocities. Then $h_0 = p_0 + \rho u_0^2/2$ becomes equal to g_0, and one obtains

$$D_i = \int_{S_0} [g_0 + (\rho u_0^2/2)] \cdot dS - \int_{S_1} [g_{1i} + (\rho u_{1i}^2/2) - (\rho/2)(v_{1i}^2 + w_{1i}^2)] \cdot dS$$

or

$$D_i = \int_{S_1} (\rho/2)(v_{1i}^2 + w_{1i}^2) \cdot dS + \int_{S_1} (\rho/2)(u_0^2 - u_{1i}^2) \cdot dS \qquad (2b)$$

or substituting in (3a),

$$D_i = \int_{S_1} (\rho/2)(v_{1i}^2 + w_{1i}^2) \cdot dS - \int_{S_1} (\rho/2)(u_0 - u_{1i})^2 \cdot dS \qquad (3b)$$

In the limiting case where the surface S_1 is at an infinite distance behind the body which shall be denoted by subscripts ∞, $u_{\infty i} = u_0$ (assuming straight trailing vortices) and

$$D_i = \int_{S_\infty} (p_0 - p_{\infty i}) \cdot dS = \int_{S_\infty} (\rho/2)(v_{\infty i}^2 + w_{\infty i}^2) \cdot dS \qquad (4)$$

This equation shows that the induced drag is transmitted as a pressure difference between the front face and the rear face of the control surface.

Expression for Profile Drag

The assumption made when considering the flow of a real fluid is that the induced drag is still given by the same formula as for an ideal fluid and that the remaining portion of the drag is profile drag. It is assumed that $v_1 = v_{1i}$ and $w_1 = w_{1i}$; however, it remains to be checked by experiments whether $p_1 = p_{1i}$.

This leads to the following expressions for the profile drag, $D_p = D - D_i$;

From Eqs. (2) and (2a):

$$D_p = \int_{S_1} (p_{1i} - p_1) \cdot dS + \int_{S_1} \rho(u_{1i}^2 - u_1^2) \cdot dS - \int_{S_1} \rho u_0(u_0 - u_1) \cdot dS \qquad (5)$$

or using Eq. (1a)

$$D_p = \int_{S_1} (p_{1i} - p_1) \cdot dS + \int_{S_1} \rho(u_{1i}^2 - u_1^2) \cdot dS - \int_{S_1} \rho u_0(u_{1i} - u_1) \cdot dS \qquad (6)$$

Since $g_0 = g_{1i}$

$$\int_{S_1} (g_0 - g_1) \cdot dS = \int_{S_1} [p_{1i} + (\rho/2)(u_{1i}^2 + v_{1i}^2 + w_{1i}^2)] \cdot dS - \int_{S_1} [p_1 + (\rho/2)(u_1^2 + v_1^2 + w_1^2)] \cdot dS = \int_{S_1} (p_{1i} - p_1) \cdot dS + \int_{S_1} (\rho/2)(u_{1i}^2 - u_1^2) \cdot dS$$

Everywhere outside the wake $g_0 = g_1$ and thus

$$\int_{S_1} (g_0 - g_1) \cdot dS = \int_{W_1} (g_0 - g_1) \cdot dS$$

where W_1 indicates the part of S_1 covered by the wake.

Thus the above formulae transform into

$$D_p = \int_{W_1} 2(g_0 - g_1) \cdot dS - \int_{S_1} \rho u_0(u_0 - u_1) \cdot dS - \int_{S_1} (p_{1i} - p_1) \cdot dS \qquad (7)$$

and

$$D_p = \int_{W_1} 2(g_0 - g_1) \cdot dS - \int_{S_1} \rho u_0 \cdot (u_{1i} - u_1) \cdot dS - \int_{S_1} (p_{1i} - p_1) \cdot dS \qquad (8)$$

or

$$D_p = \int_{W_1} (g_0 - g_1) \cdot dS + \int_{S_1} (\rho/2)[(u_{1i}^2 - u_1^2) - 2u_0(u_{1i} - u_1)] \cdot dS \quad (9)$$

In the case where the rear control surface is infinitely far behind the body, Eqs. (3) and (4) give:

$$D_p = \int_{S_\infty} \rho u_\infty (u_0 - u_\infty) \cdot dS + \int_{S_\infty} (p_{\infty i} - p_\infty) \cdot dS \quad (10)$$

A formula analogous to Eq. (9) was first proposed by Betz[1] in 1925. For the second integral in Eq. (9) he carried out the integration only over W_1, however, and thus implicitly made the assumption that the integral

$$I = \int_{S_1 - W_1} (\rho/2)[(u_{1i}^2 - u_1^2) - 2u_0(u_{1i} - u_1)] \cdot dS$$

has a negligible value. Write

$$u_{1i} = u_0 + \Delta u_v + \Delta u_d$$
$$u_1 = u_0 + \Delta u_v + \Delta u_d + \Delta u_s$$

where

Δu_v = increase in u-velocity due to equivalent vortex

Δu_d = increase in u-velocity due to equivalent doublet system representing the body.

Δu_s = increase in u-velocity due to equivalent source which gives potential flow about a body having the form of the wake.

Then

$$I = \int_{S_1 - W_1} (\rho/2) \cdot \Delta u_s \cdot (2\Delta u_v + \Delta u_s + 2\Delta u_d) \cdot dS$$

When the angle of inclination of the wing is small, the integral $\int_{S_1 - W_1} \Delta u_s \cdot \Delta u_v \cdot dS$ is approximately zero by symmetry, since Δu_v is equal and opposite above and below the wake, while Δu_s has the same values. The integral $\int_{S_1 - W_1} \rho \cdot \Delta u_s \cdot \Delta u_d \cdot dS$ can be evaluated for any shape body by calculating for the corresponding doublet and source distributions. A rough estimate of the magnitude of this factor was made for a Rankine Oval. This gave the result that

$$\int_{S_1 - W_1} \rho \cdot \Delta u_s \cdot \Delta u_d \cdot dS = \Delta C_{D_1} \cdot \rho u_0^2/2 \cdot c$$

where

$$\Delta C_{D_1} = \frac{2y_0}{c} \cdot \frac{2}{\pi} \cdot \frac{t}{c} \cdot \frac{\left(1 - \frac{1}{\pi}\frac{t}{c}\right)}{\left(1 - \frac{1}{\pi}\frac{t}{c}\right)^2 - 16\left(\frac{1}{2} + \frac{x}{c}\right)^2}$$

$2y_0$ = width of wake

t = thickness of body

TABLE 1

x/c	ΔC_{D_1}	ΔC_{D_2}	$\Delta C_{D_1} + \Delta C_{D_2}$
0	−.00124	+.0002	−.00104
1/4	−.00047	+.00013	−.00034
1/2	−.00025	+.0001	−.00015
1	−.00011	+.00007	−.00004

c = chord of body

x = distance to section S_1 from trailing edge

When similarly as above $\int_{S_1 - W_1} (\rho/2) \cdot \Delta u_s^2 \cdot dS$ is calculated, substituting a single source at the center of the body, a second correction factor to the drag coefficient is found

$$\Delta C_{D_2} = \frac{1}{8\pi} \frac{(2v_0/c)^2}{(1/2) + (x/c)}$$

A numerical calculation for the case $2y_0/c = 1/20$, $t/c = 1/8$ gave the results shown in Table 1.

These show that when measurements are made closer than about 1 chord-length behind the trailing edge of the body, the above correction factors should be applied.* The numerical values should probably be recalculated where high accuracy is required for the particular airfoil shape used rather than using the above values derived on the basis of a Rankine Oval. Since an airfoil is not so full at the rear as the oval, somewhat smaller values would be expected.

Jones[2] deduced a working formula† for the profile drag starting his analysis with a modified form of Eq. (10), namely

$$D_p = \int_{S_\infty} \rho u_\infty \cdot (u_0 - u_\infty) \cdot dS$$

i.e., he assumes $p_{\infty i} = p_\infty$. He transformed the above equation, making use of the fact that $\rho u_\infty \cdot dS_\infty = \rho u_1 \cdot dS_1 = dm$ is an element of mass which has suffered a loss in velocity $(u_0 - u_\infty)$ in unit time. Thus he writes

$$D_p = \int_{S_1} \rho u_1 \cdot (u_0 - u_\infty) \cdot dS \quad (11)$$

* Since the submission of this paper a comment on the same subject by Muttray has appeared in the "Jahrbuch 1937 der Deutschen Luftfahrtforschung," pp. 1–66. He states that the measured profile drags were found too high by about 12 percent at the trailing edge, by about 8 percent at a distance $x/c = .10$ behind the trailing edge (compared to measurements a great distance behind the wing). In every case of a wing with finite thickness the Betz formula gave a variation of the profile drag with distance from the wing. Only in the case of a flat surface consisting of thin pieces of sandpaper glued back to back was the profile drag found to be independent of distance behind the wing. These results confirm the above conclusions very closely.

† The same formula has also been deduced by Krämer and Doetsch, and had already previously been stated by Betz in 1931.

where it is understood that $(u_0 - u_\infty)$ is associated with the particle which at infinity becomes $\rho u_\infty dS_\infty$. In order to determine u_∞, Jones makes the assumption that there is a hypothetical flow in which the total energy associated with a given particle remains constant in the wake in a similar fashion as it does for an ideal fluid with vortex flow. This is not entirely correct since actually there is a small amount of dissipation of energy in the wake. The closer the plane of measurement S_1 is, however, to the trailing edge of the body, the less is likely to be the error. To get the final formula it is thus assumed that $g_1 = g_\infty$, also $p_\infty = p_{\infty i}$, $u_0 = u_{\infty i}$, $v_\infty = v_{\infty i}$, and $w_\infty = w_{\infty i}$. Also for the ideal fluid $g_{\infty i} = g_0$ and thus

$$g_1 = p_\infty + (\rho/2)[u_\infty^2 + v_\infty^2 + w_\infty^2]$$

$$g_0 = p_0 + (\rho u_0^2/2) = p_{\infty i} + (\rho/2)[u_{\infty i}^2 + v_{\infty i}^2 + w_{\infty i}^2]$$

Thus $$g_1 = p_0 + (\rho u_\infty^2/2)$$

and $$u_\infty = \sqrt{2/\rho}\,\sqrt{g_1 - p_0}$$

$$u_1 = \sqrt{2/\rho}\,\sqrt{h_1 - p_1} \text{ where } h_1 = g_1 - \frac{1}{2}\rho(v_1^2 + w_1^2)$$

Eq. (11) thus becomes

$$D_p = \int_{S_1} 2\,\sqrt{h_1 - p_1}\,(\sqrt{g_0 - p_0} - \sqrt{g_1 - p_0})\cdot dS$$

Since everywhere outside the wake $g_1 = g_0$ this formula reduces to

$$D_p = \int_{W_1} 2\,\sqrt{h_1 - p_1}(\sqrt{g_0 - p_0} - \sqrt{g_1 - p_0})\cdot dS \tag{12}$$

This formula thus has the great advantage over Betz's that it should hold well when applied close to the trailing edge of the body without requiring any theoretical correction factors. It is to be pointed out that the surface W_1 should really be taken perpendicular to the free stream velocity and not perpendicular to the plane of the wing as Jones asserts. The two cases differ, however, only by a factor $(1 - \cos\alpha)\cdot D_p$ where α is the angle of attack.

The working formula corresponding to Eq. (7) or its equivalent forms Eqs. (8) or (9) becomes

$$D_p = \int_{W_1} 2(g_0 - g_1)\cdot dS - \int_{W_1} 2\,\sqrt{g_0 - p_0}(\sqrt{g_0 - p_0} - \sqrt{h_1 - p_1})\cdot dS + \Delta D_p \tag{13}$$

where ΔD_p is a small correction factor which requires some theoretical calculations:

$$\Delta D_p = -\int_{W_1}(p_{1i} - p_1)\cdot dS + \int_{S_1 - W_1}(\rho/2)\cdot \Delta u_s \cdot \times (\Delta u_s + 2\Delta u_d)\cdot dS$$

Krämer and Doetsch[3] in their experiments at the D.V.L. simply assumed a formula of the type

$$D_p = K\cdot\int_{W_1}(g_0 - g_1)\cdot dS$$

They determined the mean value of the above integral by means of a whole array of total head tubes which were connected together so as to give the integrated value as a single reading. The constant K was determined experimentally as a function of distance behind the wing. The above method requires still a considerable amount of further investigation to prove that K is actually a universal function, independent of wing thickness and angle of attack.

The main difficulties with the experimental determination of the quantities required in working Eqs. (12) and (13) is first of all that the turbulence of the wake has a considerable effect especially upon the static pressure determination, as shown by the experiments of Schlichting.[4] Secondly, in addition to the total head $g_1 = p_1 + (\rho/2)(u_1^2 + v_1^2 + w_1^2)$, the quantity $h_1 = p_1 + (\rho u_1^2/2)$ is required as shown by Eqs. (12) and (13). The error caused by making the survey with a single total head tube to serve for both readings should be investigated, especially at the higher angles of attack when the quantity $(\rho/2)(v_1^2 + w_1^2)$ assumes a larger value. Other sources of error such as the effective displacement of the center of a pitot tube in a pressure gradient have been investigated by Jones.

Summary and Results

Formulae for the parasite drag from wake measurements corresponding to those by Betz and Jones have been deduced from basic principles. It is pointed out that Betz neglected quadratic terms of small velocities which, when measurements are carried out very close to the body may introduce appreciable errors. Jones's formula does not suffer from this defect; in fact it should hold best close to the body before the wake has suffered much energy dissipation.

Both Jones and Betz neglected the quantity $(\rho/2)\cdot(v_1^2 + w_1^2)$ in their formulae as small, which should be justified experimentally. Jones's assumption of no energy loss in the wake appears reasonably good but requires also more thorough experimental verification. It should also be verified whether the pressure in the turbulent wake is the same as that calculated from a pure potential flow. It is pointed out that the plane of measurements in the wake should be normal to the free stream velocity, otherwise a small correction should be introduced.

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Institute Notes

New Corporate Members

Five subsidiaries and the parent Bendix Aviation Corporation became Corporate Members of the Institute in March. The Lockheed Aircraft Corporation became a Corporate Member this month.

Trip of President and Secretary of the Institute

The President and the Secretary of the Institute visited Chicago, St. Louis, Minneapolis, and Los Angeles during March and April. Major Gardner returned to New York through the courtesy of the Pan American Airways *via* Mexico City, Merida, and Havana, and flew from Miami to New York through the courtesy of Eastern Air Lines. A detailed account of their trip will appear in the next issue of the Journal.

Meetings

On March 8 about thirty members of the Institute gave a luncheon in honor of George E. Woods Humphery, Managing Director of the Imperial Airways, at the University Club, New York.

On Wednesday, March 23, the Los Angeles Branch of the Institute, in cooperation with the American Society of Metals, presented a program of technical papers dealing with the use of metals in the aircraft industry. The first aeronautical session was held at the Biltmore Hotel in Los Angeles on Wednesday morning. The afternoon session was held in the Pan Pacific Auditorium. The program follows:

Morning Session—9:30 a.m.—Biltmore Hotel

Chairman

C. L. Hibert, Consolidated Aircraft Corporation

J. R. Goldstein, Douglas Aircraft Company, "Metals in the Aircraft Industry."

Arthur W. Winston, Dow Chemical Company, "Recent Developments in the Application of Magnesium Alloys to Aircraft."

A. A. Handler and L. W. Davis, Aluminum Company of America, "Design and Production of Aluminum Alloy Aircraft Forgings."

Bradley Stoughton, Lehigh University, "Engineering Alloy Steels (S. A. E. Specifications)."

Afternoon Session—2:00 p.m.—Pan Pacific Auditorium

Chairmen

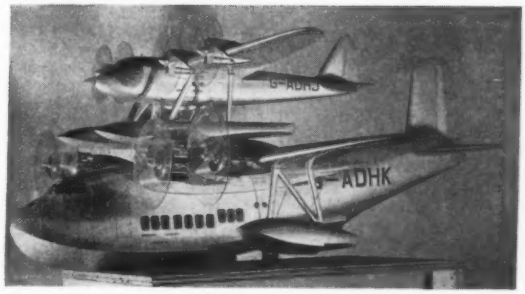
Howard D. Houghton, Douglas Aircraft Company
E. E. Sechler, California Institute of Technology

Waldemar Naujoks, Steel Improvement & Forge Company, "Steel Forgings for Aircraft."

Carl de Ganahl, Fleetwings, Inc., "Stainless Steel for Aircraft."

Tom A. Triplett, Triplett and Barton Laboratories, "X-Ray Inspection of Aeronautical Materials."

Plans are being completed by the Institute to hold national meetings in Los Angeles, Washington, and Chicago as well as in New York. Each meeting will have a special purpose and command wide interest. Members and Branches should always remember in making plans that October 15 will always be celebrated as the Anniversary of the founding of the Institute, and December 17 as the day of the Wright Brothers' Lecture and Honors Night.



The Short Mayo composite aircraft.

Gifts to the Institute

The donation of three book-cases and books by Major E. E. Aldrin is recorded on another page of the Journal with a list of the books he gave. Ernest W. Dichman also presented several hundred valuable pamphlets and reports. A. J. M. Hamon presented F. T. Miller's "The World in the Air" and "Peru from the Air" by George L. Johnson.

The Council acknowledges these donations with appreciation and hopes that other members will also wish to add to the rapidly growing library.

Dr. Lyman J. Briggs, Director of the National Bureau of Standards, has presented a glass tube containing air from the stratosphere taken by Captain A. W. Stevens at an altitude of approximately 14 miles during the flight of Explorer II on November 11, 1935.

T. P. Wright, President of the Institute, presented a rib from the famous NC-4, the first flying boat to fly the Atlantic.

G. E. Woods Humphery, Managing Director of the Imperial Airways, presented a large model of the composite aircraft designed by another Institute member, Major R. H. Mayo. Glenn L. Martin presented a large photographic mural of the China Clipper creating a whirlpool on one of her taxying tests.

Charles Marcus gave a decorative stand holding a glass ball found by the Pan American staff at Midway. These balls are used by Japanese fisherman to float their nets. During storms the nets are blown to sea and sometimes these floats float across the Pacific and are found on the coast of California.

The United Airlines presented a model of the DC-4.

Another gift has been made to the Institute which will be announced in the next issue of the Journal.

Aviation Motion Picture Annual

The Motion Picture Producers and Distributors of America, through the courtesy of Hon. Will H. Hays, are preparing for showing at Institute Meetings what will be known eventually as the Aviation Motion Picture Annual. All the aviation subjects that appear in the news films of six companies are sent each week to the Institute where they are joined and made into a continuous sound film. At the end of the year it will be edited and shown for the first time at the Seventh Annual Meeting of the Institute and later at Branch meetings. This courtesy on the part of the motion picture companies is in recognition of the great value aviation has been to the motion picture industry. This is but a preliminary announcement of the film of which members will hear more later in the year.

Fifth International Congress *for* Applied Mechanics, 1938

The Fifth International Congress for Applied Mechanics will meet in Cambridge, Mass., September 12 to 16, 1938, at Harvard University and Massachusetts Institute of Technology. The Congress is a meeting of persons working in the field of Applied Mechanics who become members and may participate in its activities by payment of the registration fee of $5.00. A program of papers to be presented by members will be organized by selection from titles, with abstracts of 300 words, in the hands of the Secretariat by July 1, 1938. Such papers are limited to 20 minutes for reading the paper and 10 minutes for discussion. Papers presented at the Congress will be published in its Proceedings.

General Lectures will be delivered by prominent scientists from England, France, Germany, Norway, and the United States on invitation of the Organizing Committee.

The program will cover three main divisions of Applied Mechanics as follows:

(1) Structures, elasticity, plasticity, fatigue, strength theory, crystal structure.

(2) Hydro and aerodynamics, gasdynamics, hydraulics, meteorology, water waves, heat transfer.

(3) Dynamics of solids, vibration and sound, friction and lubrication, wear and seizure.

Following the meeting at Cambridge, arrangements have been made to visit Washington (National Bureau of Standards), and Langley Field (National Advisory Committee for Aeronautics).

Dormitory facilities will be made available by Harvard University at a nominal charge.

A bulletin giving further details will be mailed to those who address an inquiry to the

Fifth International Congress for Applied Mechanics,
Massachusetts Institute of Technology,
Cambridge, Mass., U. S. A.

TH. VON KÁRMÁN
J. C. HUNSAKER
Secretaries

Motion Picture of Langley Field Laboratories

The motion picture prepared for showing at meetings of the Institute by the National Advisory Committee for Aeronautics is now ready to be loaned to Branches. It is accompanied by comments prepared by the N.A.C.A. and takes about one hour to show.

Aeronautical Reviews

Because of illness of the Editor of the Technical Data Digest, from which the Aeronautical Reviews published each month in the Journal are prepared, these reviews have not been available for the April issue of the Journal. It is expected, however, that enlarged issues of the Technical Data Digest will be available for publication in the May issue of the Journal.

Gift of Books to the Institute Library

The following books have been presented to the Institute Library by Major E. E. Aldrin.

The Council of the Institute thanks Major Aldrin for his generous gift of books that he has collected for many years.

(continued from March 1938 issue)

Precision of Measurements and Graphical Methods, by H. M. Goodwin; 1913.

The Dynamics of Particles and of Rigid, Elastic and Fluid Bodies, 1912. Second Edition.

Strength of Materials, by Arthur Morley; 1916.

The Theory of Electricity and Magnetism, by Arthur Gordon Webster; 1897.

Five Place Logarithmic & Trigonometric Tables, by Levi L. Conant; 1909.

A Text-Book of Physics, by A. Wilmer Duff; 1909. Second Edition.

Introduction to the Calculus of Variations, by William Elwood Byerly; 1917.

A Treatise on Hydromechanics, by A. S. Ramsey; 1913.

Calculus, by Wm. S. Franklin, Barry Macnutt, and Rollin L. Charles; 1913.

Engineering Electricity, by Ralph G. Hudson; 1920.

The Temperature-Enthropy Diagram, by Charles W. Berry; 1913.

First Course in Infinitesimal Calculus, by Daniel A. Murray; 1903.

The Thermodynamic Properties of Ammonia, by Frederick G. Keyes and Robert B. Brownlee; 1916.

Increasing Power Output of Internal Combustion Engines by Supercharging, by Dr.- Ing. W. Zinner; 1926.

Principles of Physical Chemistry, by Edward W. Washburn; 1915.

General Chemistry for Colleges, by Alexander Smith; 1911.

Practical Instructions for Using the Steam Engine Indicator, 1916.

Text-Book of Mechanics, by Louis A. Martin, Jr., 1916. Vol. VI.

A Short History of the U. S., by Edward Channing; 1900.

First Course in Algebra, by Herbert E. Hawkes, William A. Luby, and Frank C. Touton; 1909.

How to Keep Invention Records, by Harry A. Toulmin; 1920.

Plane and Spherical Trigonometry, by Levi L. Conant; 1909.

A.S.M.E. Power Test Code-Rules, 1915 edition.

Descriptive Geometry, by John C. Tracy and Herbert B. North; 1914.

Aircraft Design Data, 1920. (Vols. 1 and 2.)

Machine Design, by Geo. B. Haven; 1916.

Les Materiaux des Constructions Mecaniques et Aeronautiques, by Ed. Marcotte and E. Berehare; 1921.

The Mechanics of Engineering, by A. Jay Du Bois; 1912.

Heating and Ventilating Engineering, by Edward F. Miller, Charles F. Park, and Wm. G. Snow; 1914.

A Short Course on the Theory and Operation of the Free Balloon, by C. H. Roth; 1917. Second Edition.

Death in the Air; 1933.

Annuaire de l'Aeronautique, by J. Reimbert and M. S. Heraud; 1929.

Neuzeitliche Flugmotoren, by Dr.-Ing. Hans Katz; 1928.

Fliegerwetterkunde, by Dr. Phil Gotthard Burghardt; 1927.

This Aviation Business, by Ernest W. Dichman; 1929.

Instructions for Installation, Inspection and Maintenance of the U. S. A. Standardized Aircraft Engine, by B. V. Bassett.

Anlage und Betrieb von Luftschiffhafen, by Dipl.-Ing. Christians; 1914.

Symposium on Aircraft Materials, by Horace C. Knerr; 1930.

Taschenbuch für den Maschinenbau, by Prof. H. Dubbel; 1920. (2 Vols.)

"Hutte" Des Ingenieurs Taschenbuch, Akademischen Verein Hutte; 1915. (2 Vols.)

Winged Victory, by V. M. Yeates; 1934.

The Aviator and the Weather Bureau, by Ford A. Carpenter; 1917.

Gli Aeroporti Doganali Italiani; 1927.

Jump, by Don Glassman; 1930.

The War in the Air, by H. A. Jones; 1928. (Vol. 2.)

Die Deutschen Luftstreitkrafte im Weltkriege, by Georg Paul Neumann; 1920.

Chemie der Gase, by Dr. Friedrich Brahmer; 1911.

Aviation Industry, The Curtis Publishing Co., 1930.

Resumes des Memoires Presentes, V Congres International de la Navigational Aerienne; 1930.

La Grande Croisiere de Costes et Le Brix, by Jean Hallo.

Aviacon y Mecanica, by Mario Torres Menier and Dr. Felipe Prieto Y. Facciolo; 1932.

Der 1000 PS Flugmotor, by Dr.-Ing. Edmund Rumpler; 1921.

American Airport Designs; 1930.

Jahresbericht, by Dr.-Ing. Friedrich Seewald; 1931.

Die Luftverkehrswertschaft in Europa und in Den Vereinigten Staaten von Amerika, by Prof. Dr.-Ing. Carl Pirath; 1931.

Flugmotoren, by Konrad Muller; 1918.

Aircraft Year Book. (18 Vols.)

Technical Reports, British Advisory Comm. for Aeronautics, 1909–1917. (6 Vols.)

Journal of the Royal Aeronautical Society, 1919–1934. (16 Vols.)

Smithsonian Misc. Collections-Bibliography of Aeronautics, by Paul Brockett; 1910.

N.A.C.A. Bibliography of Aeronautics, 1909–1932.

Bulletin of the Airplane Engineering Dept., U.S.A., (Vols. 1 to 4.)

Bulletin of Experimental Dept. Airplane Engineering Division, U. S. A., (Vols. 1 to 4.)

Air Service Information Circulars. (6 Vols.)

Index to Reports of Materiel Division, U. S. Army Air Corps, January, 1927.

First Annual Report of the N.A.C.A., 1915.

Third Annual Report of the N.A.C.A., 1917. (Parts 1 and 2.)

N.A.C.A. Reports, 1917–1932.

Reports on Wind Tunnel Experiments in Aerodynamics, by J. C. HUNSAKER, E. BUCKINGHAM, H. E. ROSELL, D. W. DOUGLAS, C. L. BRAND, and E. B. WILSON; 1916.

Journal of the Franklin Institute, 1914.

Rapports—Premier Congres International de la Securite Aerienne, 1932–1933. (Vols. 1–5.)

Discussions et Voeux—Premier Congres International de la Securite Aerienne 1934. Vol. 5. (2 copies)

Technical Bulletin No. 48, Air Service Information Circular; Vol. 5, No. 490, 1926.

Yearbook for 1926.

S.A.E. Transactions; Vols. 22, 23, 24, 25, 26, 27, 29, 31.

A Handbook of Colloquial Japanese, by BASIL HALL CHAMBERLAIN; 1907. Revised 4th Edition.

Transactions, The American Society of Mechanical Engineers; 1919. (Vols. 41–46.)

Record and Index, The American Society of Mechanical Engineers; 1929. (Vol. 3.)

Preprinted Papers & Program, Aeronautic and Hydraulic Divisions American Society of Mechanical Engines; 1934.

Index of References—Aeronautica—Applied Subjects of Construction, 1920.

Mechanical Catalog 1936–1937, 1936.

Market Data Handbook of U. S., by PAUL W. STEWART; 1929.

Airports & Established Landing Fields in the U. S., The Airport Directory Co., 1933.

Preprints of Technical Papers Delivered before the Royal Aeronautical Society, London. (2 Vols.)

Financial Handbook of tne American Aviation Industry. 1929.

Translated Abstracts of Technische Berichte, 1917, 1925. (2 Vols.)

Aircraft Manual.

Instruction Book for Axelson Aircraft Engine Type B.

Handbook, Curtiss Standard Jn4-D Military Tractor; 1918.

Guia Aeronautica, 1934.

Cinquieme Congres International de la Navigation Aerienne, 1931.

Air Ministry Report for the Years 1935–1936.

Report of the N. Y. State Commission on Aviation, 1930.

First National Aeronautical Safety Conference, The Daniel Guggenheim Fund for the Promotion of Aeronautics and the National Safety Council; 1928. (2 copies.)

Report of Bureau of Aeronautics, State of Ohio, 1929.

A Study of Transportation by Airway as Related to Competition with Rail Carriers in Continental U. S., 1932.

Bulletin of Information No. 462, International Commission for Air Navigation, June, 1931.

Instruction Book for Wright Cyclone Engines, 1930, (Second Edition).

Instruction Book for Wright Tornado Engines, 1927.

Instruction Book for Wright Whirlwind Engines, 1928.

Instruction Book for Wright Gipsy Engines, 1929.

Instruction Book for Wright Typhoon Engines, 1927.

Instruction Book for the Packard-Diesel Aircraft Engine, 1931.

Miscellaneous Collections Aeronautica, by J. C. HUNSAKER, RICHARDSON, and OTHERS.

Deutsche Luftfahrt, by EUGEN ZABEL; 1918.

Kuhlung und Kuhler für Flugmotoren, by DR.-ING. PULZ; 1920.

Book Reviews

History of Aeronautics, compiled by WILLIAM B. GAMBLE; The New York Public Library, 1938; 325 pages, $2.00.

This is the most useful bibliography of books, articles, and pamphlets dealing with the history of aeronautics that has been published. The author is Chief of the Science and Technology Division of the New York Public Library, and has spent four years compiling references to the aeronautical collection of over 5000 volumes in the library. While the book gives references to outstanding events that occured after the Wright Brothers' first flight in 1903, it is primarily concerned with every event before that epoch making flight.

There are many bibliographies on aeronautics but most of them presume that the enquirer knows the name of a book or its author. Mr. Gamble has arranged his book of 325 pages so that anyone wishing to secure information on any one of the many subjects that comprise aeronautical history may do so without much previous knowledge of the subject.

In addition to a careful division of subjects, short suggestive comment on almost every book or article gives the reader just the pertinent information he needs to select a reference.

Even if the book had ended on page 254 it would have been of inestimable value, but the outstanding contribution of the author is the 71 pages of indexes. He gives complete references to all authors and subjects in the bibliography. But he goes much further. Under Bird Flight, for instance, a special bibliography is given and then the author lists all those experimenters who have written on this subject. Under Bombing Apparatus will be found a list of some of the earliest experiments in this field.

It is extraordinary to find a librarian, who had no special interest in aviation, preparing such an exhaustive and helpful reference work as a part of his professional work.

Every aeronautical writer, editor, and investigator will be greatly indebted to Mr. Gamble for making such a valuable contribution to aeronautical literature.

LESTER D. GARDNER

Jahrbuch der Deutschen Akademie der Luftfahrtforschung 1937-1938; Deutsche Akademie der Luftfahrtforschung, Berlin, 1938; 321 pages.

One of the most perfect examples of fine book making is this Yearbook of the Academy of Aeronautical Research containing biographies of all the distinguished members. The pictures of the members are reproduced in gravure and the biographies are an invaluable reference for securing information regarding many of the leading aeronautical scientists of the world.

The Academy consists of only the most distinguished aeronautical specialists in Germany and through careful selection includes as members a few leaders from many other countries. The care with which such selections have been made will be apparent from the list of American members which has been limited to Dr. Ames, Dr. Briggs, Dr. Durand, Dr. Hunsaker, Dr. Lewis, and Dr. Millikan.

Another feature of the book is a list of all the papers that have been written by each member. The concluding chapters give the proceedings of the meeting of the Council held in April, 1937. This book was presented to the Institute by Ministerialrat A. Baeumker, who is Chancellor of the Academy, and it will make a valuable addition to the library of the Institute.

SUGGESTIONS FOR CONTRIBUTORS TO THE JOURNAL *of the* AERONAUTICAL SCIENCES

The Institute of the Aeronautical Sciences invites both members and non-members from any country to submit papers for publication in the Journal of the Aeronautical Sciences. The Journal, following the practice of other scientific publications, does not pay for contributions.

The following directions for the preparation of papers, if followed by authors, will save correspondence, avoid the return of papers for changes, minimize the work of preparation for the printer, and save the expense due to the charges made for "author's corrections."

MANUSCRIPTS: Papers must be written in English, in original typewriting on one side only of white paper sheets, consecutively numbered, and be double or triple spaced with wide margins. Manuscripts should be prepared with great care so that they will be typographically accurate. Paragraphing should be given special attention. Papers should be written in the third person, reference to the writer being made as "the author." Avoid the use of the words "I," "we," and "you." Blueprint copies of papers are unacceptable as it is impossible to mark directions to the printer on them. Correcting, changing, or adding to papers after they are in type is costly. It is, therefore, imperative that papers submitted be in final form. Typographical errors may be corrected on proofs, but if authors wish to add material, they may do so at their own expense. In mailing, drawings may be rolled, but manuscripts should be sent flat. Send by first class mail (register if you wish for your own protection) to the Secretary, Institute of the Aeronautical Sciences, 5111 RCA Bldg., Rockefeller Center, New York City. All manuscripts will be examined by the Editorial Committee and by the Editor. Authors will be advised as promptly as possible (usually two to three weeks) whether the paper is acceptable for publication.

TITLES: The title of the paper should be brief. The name and initials of the author should be written as he prefers. The use of the full name of an author is advocated because of the frequent duplication of initials and surnames which sometimes makes it difficult to establish the identity of the author. This is particularly important for large annual indexing and abstracting services. All titles and degrees or honors are omitted. The name of the organization with which the author is associated should be placed after his name on the same line. The date on which the paper is received will be inserted by the Editor.

SUMMARIES OR ABSTRACTS: An abstract to be printed at the beginning should accompany each article. It should be adequate as an *index* and as a *summary*. It should contain a statement of major conclusions reached, since summaries in many cases constitute the only source of information used in compiling scientific reference indexes. Abstracts printed in other journals, especially foreign, in most cases, consist of summaries from printed papers. The summary should explain as adequately as possible the major conclusions to a non-specialist in the subject. The summary should contain from 100 to 300 words, depending on the length of the paper.

SUB-HEADINGS: Sub-headings should be inserted by the author at frequent intervals. The work of editorial preparation will be simplified by the author providing many sub-headings. Owing to the breaking of columns and the insertion of illustrations, some of the sub-headings may have to be omitted.

SHORTENING OF PAPERS: Some papers, at the end, fill in only a portion of a page. This leaves much wasted blank space as succeeding articles are started at the top of a page. Authors should indicate by notation on the left-hand side of the page what matter may be omitted when "run overs" occur. This request is important as the Journal cannot afford in the future, as it has in its earlier issues, to have blank half pages or more at the end of papers.

MATTER USUALLY DELETED: Acknowledgments of assistance in preparation of paper, except by collaborators. Photographs or illustrations of little technical interest and not showing advances in general practice. Too detailed tabular matter (general results of such tables may be included in the text). Lengthy descriptions of materials or processes or of preliminary experiments or theories which preceded final results; salient features only are of interest.

REFERENCES AND FOOTNOTES: References should appear as footnotes only, numbered consecutively, grouped together at the end of the manuscript. The arrangement should be as follows; (for books)—[1] Durand, W. F., *Aerodynamic Theory*, Vol. 1, p. 23; Julius Springer, Berlin, 1934. (For magazines)— [1]Englund, C. R., Crawford, A. B., and Mumford, W. W., *Some Results of a Study of Ultra-Short-Wave Transmission Phenomenon*, Proc. I. R. E., Vol. 20, No. 12, pp. 481 and 482, March, 1933. Please give Author, Title, Volume, page, publisher and date of publication as indicated. Omission of one required fact causes much extra editorial work and possible inaccuracies. All references are grouped at the end of the article

ILLUSTRATIONS: Illustrations should accompany manuscripts and each should always be referred to in the text, preferably by number. Drawings or graphs should not be larger than 12×16 inches, and must be made with jet black India Ink on white paper or tracing cloth, the latter being preferred. *Do not use typewriter for lettering.* The smallest lettering on 8×10 inch figures should be no less than $^1/_4$ inch high. Cross-section paper (white with black lines) may be used, but should not have more than 4 lines per inch. If finer ruled paper is used, the major division lines should be drawn in with black ink, omitting the finer divisions. In the case of finely ruled paper only blue-lined paper can be accepted. *Tracing paper and blueprints* are not acceptable. Lettering and all markings must be large enough to be readable after reduction. Mail rolled or flat, never fold. Drawings which cannot be reproduced (including pencil drawings) will be returned to the author for redrawing, thus delaying publication of the paper. Photographs should be very distinct and show clear black and white contrasts. They must be on glossy white paper. Avoid round and oval photographs.

CAPTIONS AND LEGENDS: Legends or captions must accompany each drawing or photograph submitted. If written on the drawing or photograph, they should be placed below and well outside the part to be reproduced. It is better to place them on separate sheets of paper pasted to the back of the drawings or photographs. Each table should have a caption such as Table 1, Table 2, Table 3, etc. Captions should be complete in themselves so as to make the data intelligible to the reader without reference to the text. A duplicate list of captions for figures should be included as the last page of the manuscript. Use "Fig. 1" (not Figure 1), Figs. 3 and 4, etc., in both the text and the numbering of illustrations. In the text, "Eq. (1)," or "Eqs. (1) and (2)" are preferable to "Equation (1)." In captions and legends, except for "Fig." and "Eq." and table headings, write all words in full; do not abbreviate. Avoid placing explanatory written matter in the drawings; it should be in the text.

MATHEMATICAL WORK: Only the very simplest formulae should be typewritten; all others should be very carefully written in pen and ink, the writing to be large enough so that ample room is provided to mark mathematical matter for the printer. A considerable space for marking should be allowed above and below all equations. All complicated equations should be repeated on separate sheets with plenty of space left for marking. The solidus should be used for simple fractions appearing within the text. Make all expressions clear to the typesetter. Greek letters used in formulae should be clearly designated by name on the margin of the manuscript. All symbols should be clearly written and carefully checked. The difference between capital and lower-case letters should be clearly distinguished and care taken to avoid confusion between zero (0) and the letter (o), between the numeral (one) and the letter (ell) and the prime ('), between alpha and a, kappa and k, u and mu, v and nu, n and eta. All subscripts and exponents should be clearly marked and dots and bars over letters or mathematical expressions should be avoided. Avoid complicated exponents and subscripts, When it is necessary to repeat a complicated expression, it should be represented by some convenient symbol.

NOMENCLATURE AND ABBREVIATIONS: The National Advisory Committee for Aeronautics Nomenclature should be used in preference to any others. Standard abbreviations should be used, and it should be noted that most abbreviations are lower case, such as m.p.h., b.m.e.p., i.hp., b.hp., hp., . . . etc.

INSTITUTE *of the* AERONAUTICAL SCIENCES

AIMS

The Institute of the Aeronautical Sciences is organized to promote the application of science in the development of aircraft. By grading members according to their qualifications, recognition is given to outstanding professional work and an incentive is provided younger members for the investigation of new aeronautical problems. The presentation and discussion of technical papers at Institute meetings afford members opportunities to hear and meet the leaders in the various aeronautical sciences. Through the Journal of the Aeronautical Sciences, members are provided with an opportunity to publish and to read of new knowledge and new applications in the field of aeronautical engineering. By including in its membership the leading aeronautical specialists in other countries an interchange of international thought is made possible. The continuous improvement of aircraft depends on scientific research and engineering experiment applied to design. The Institute desires to bring together all specialists contributing to aeronautical progress and, for this purpose, is keeping the dues nominal in amount, in view of the fact that practically every specialist already belongs to his own special professional society. The dues from members do not provide funds enough both for publication of the Journal and for the Institute's other activities, the difference being made up from funds contributed by benefactors and by the aeronautical industry.

THE SKYPORT, *the headquarters of the Institute, located on the fifty-first floor of the largest office building in the world, provides members when in New York with a convenient and attractive meeting place.*

DUES

The entrance fee for all members is five dollars. The annual dues are: Fellows and Associate Fellows, $10.00; Members, $10.00; Pilot Members, $10.00; Industrial Members, $10.00; Technical Members, $5.00.

APPLICATION for MEMBERSHIP

Application for membership forms may be secured from any member of the Institute or from the Secretary. An Applicant is required to furnish the names of several members acquainted with his professional work. The Admissions Committee recommends the grade of membership to the Council which, in its discretion, may extend to the Applicant an invitation to join the Institute.

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